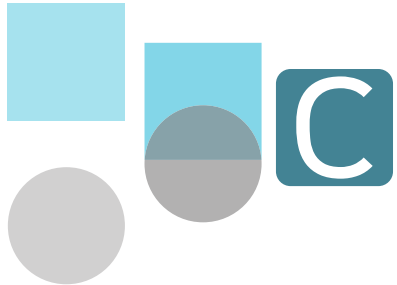


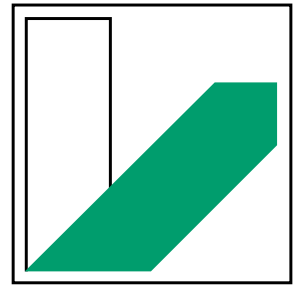

Enumerating Pattern Matches in Texts and Trees

Antoine Amarilli[1], Pierre Bourhis[2], Stefan Mengel[3], Matthias Niewerth[4]

October 24th, 2018

[1]Télécom ParisTech

[2]CNRS CRIStAL

[3]CNRS CRIL

[4]Universität Bayreuth

Problem: Finding Patterns in Text

- We have a long text T:

```
Antoine Amarilli Description Name Antoine Amarilli. Handle: a3nm. Identity Born 1990-02-07.
French national. Appearance as of 2017. Auth OpenPGP. OpenId. Bitcoin. Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France. Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016. Former student of the École normale supérieure.
More Résumé Location Other sites Blogging: a3nm.net/blog Git: a3nm.net/git ...
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    - Write the pattern as a regular expression:

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→ How to find the pattern P efficiently in the text T?

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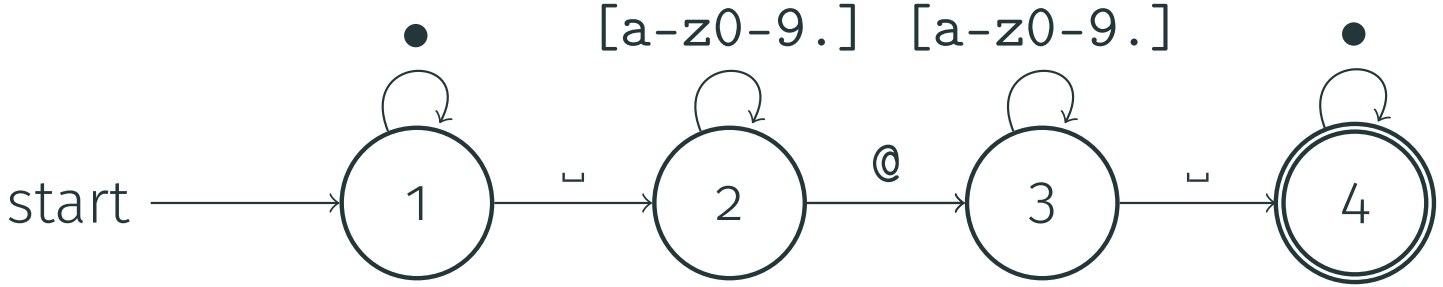

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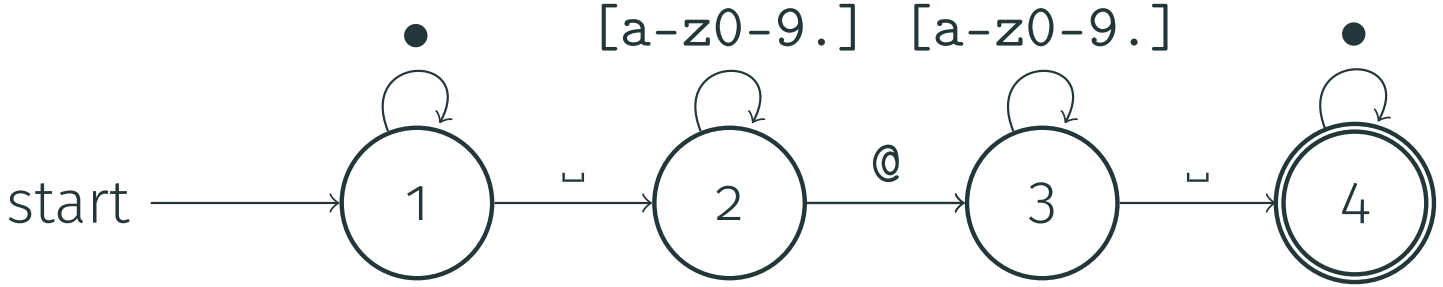

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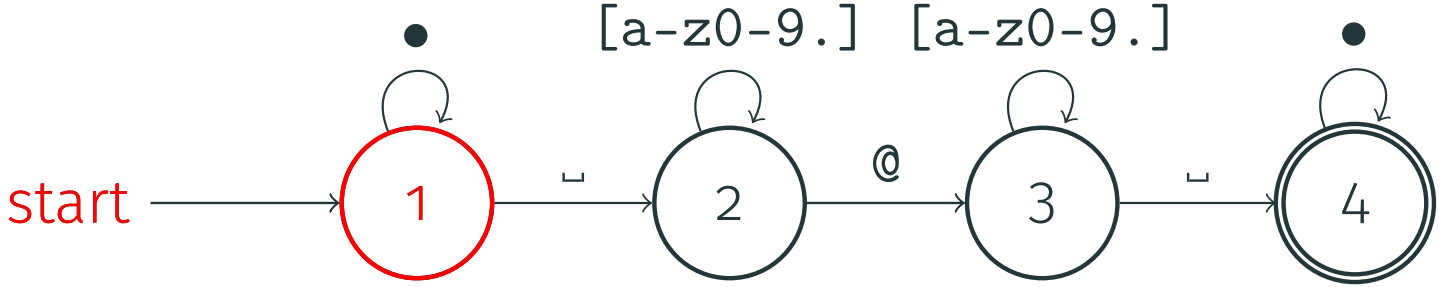

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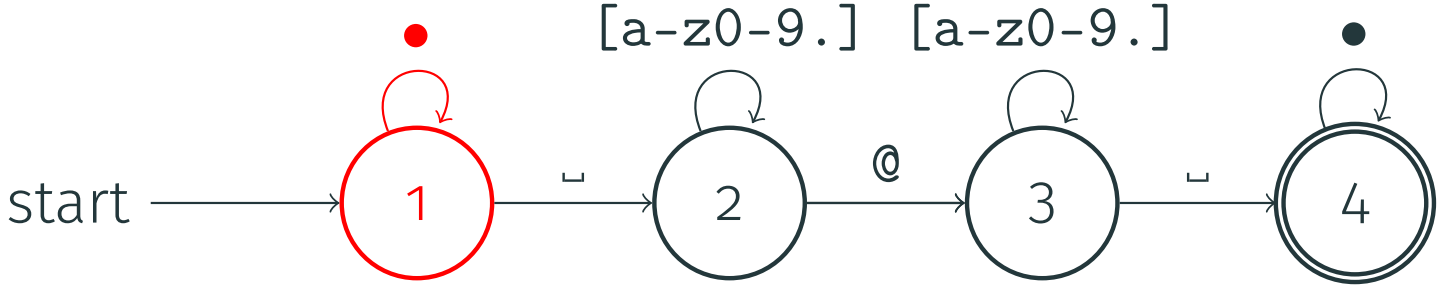

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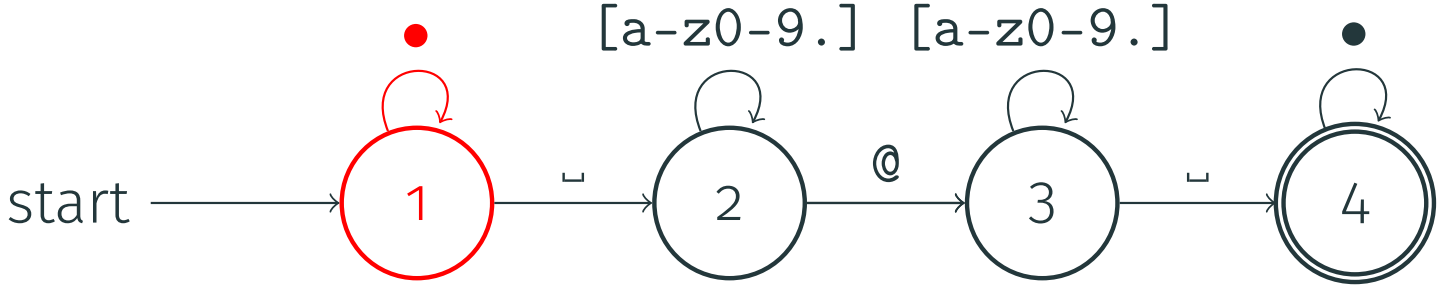

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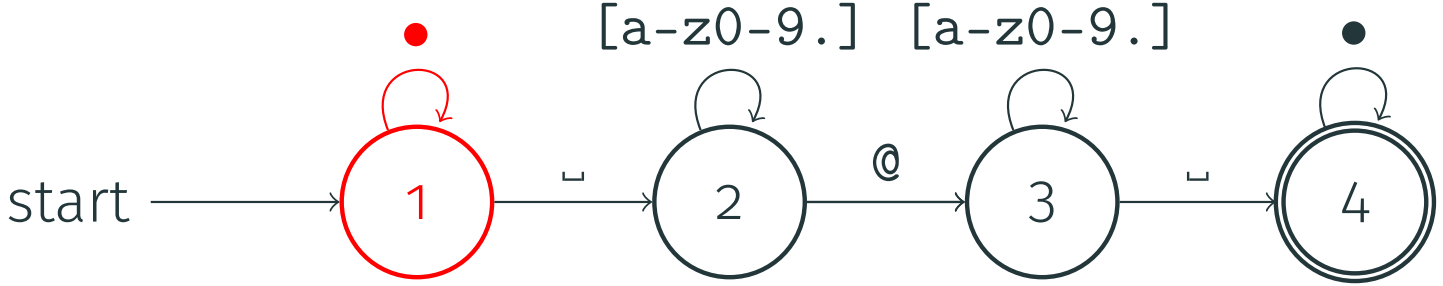

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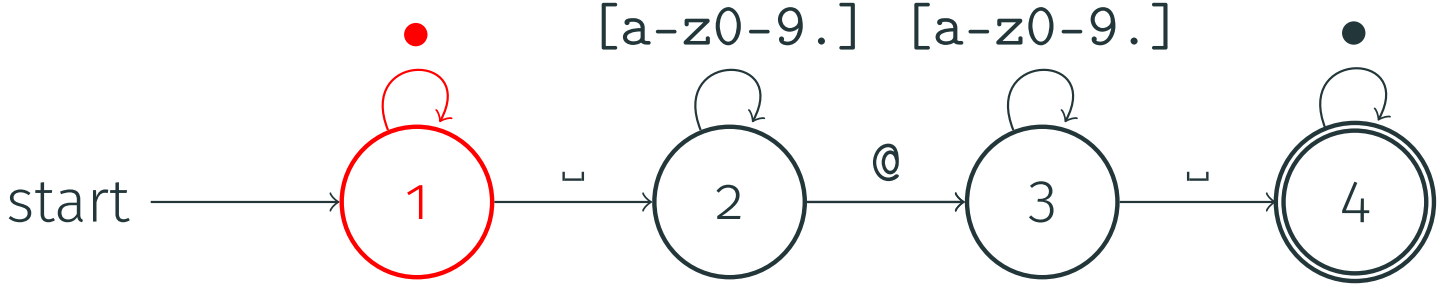

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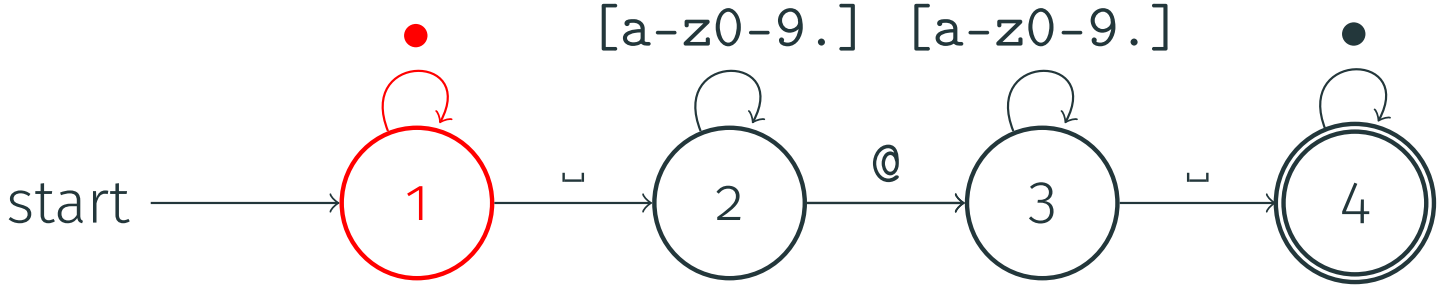

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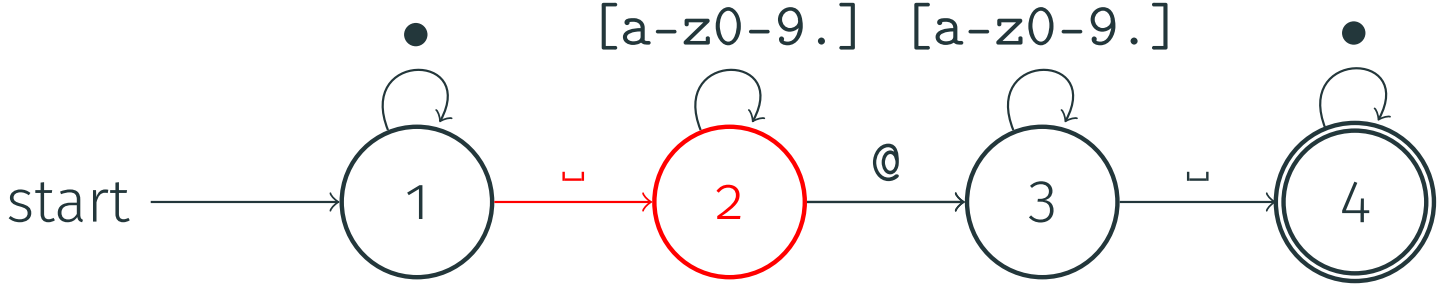

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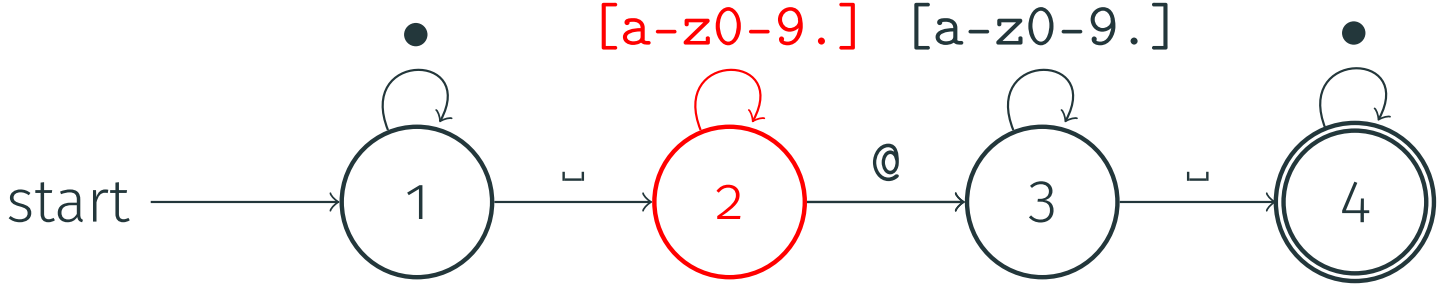

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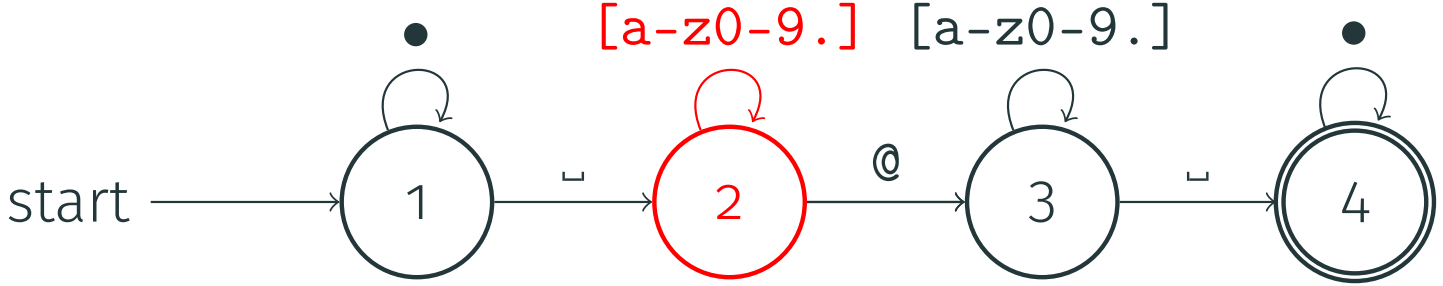

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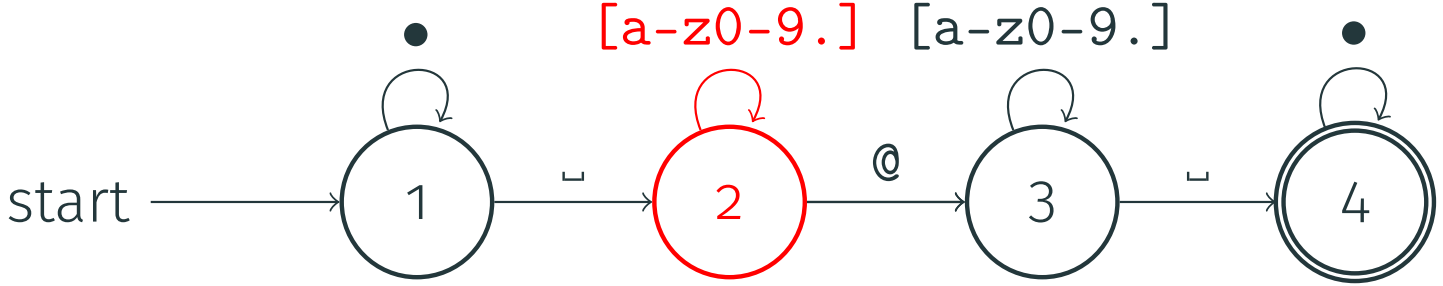

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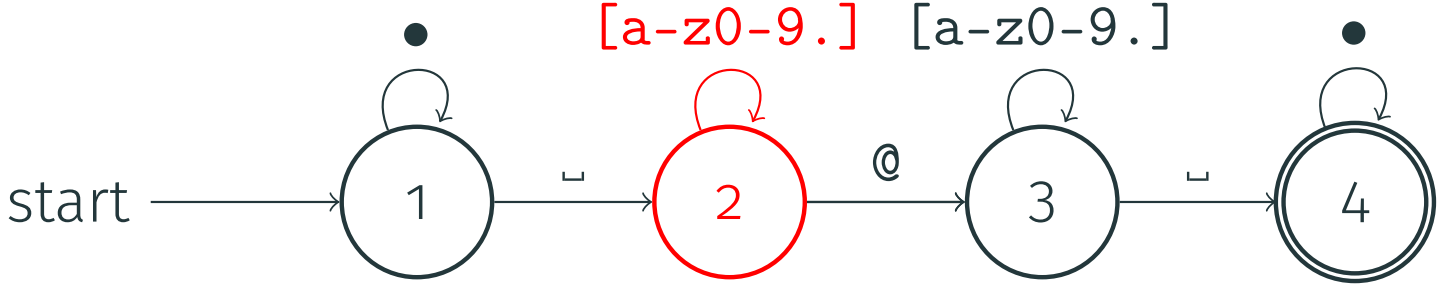

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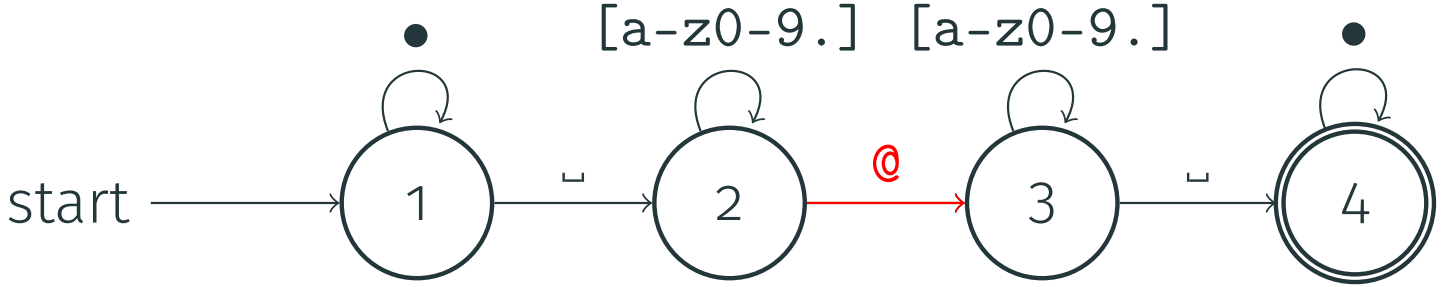

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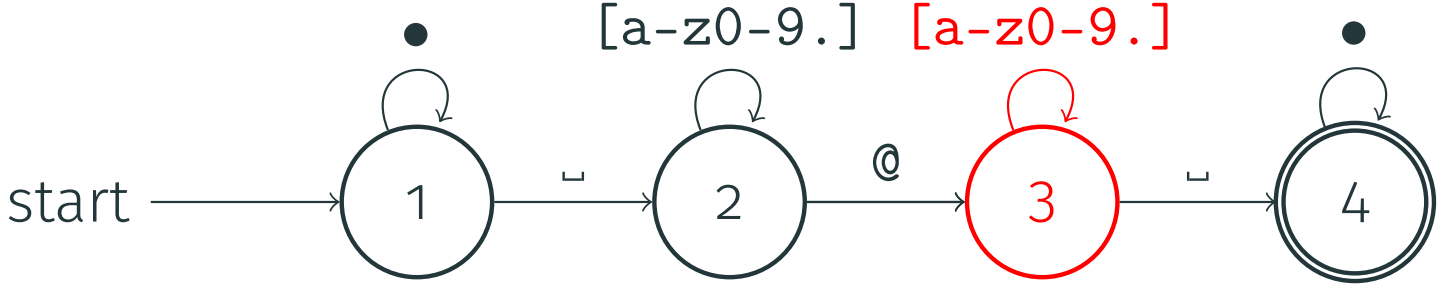

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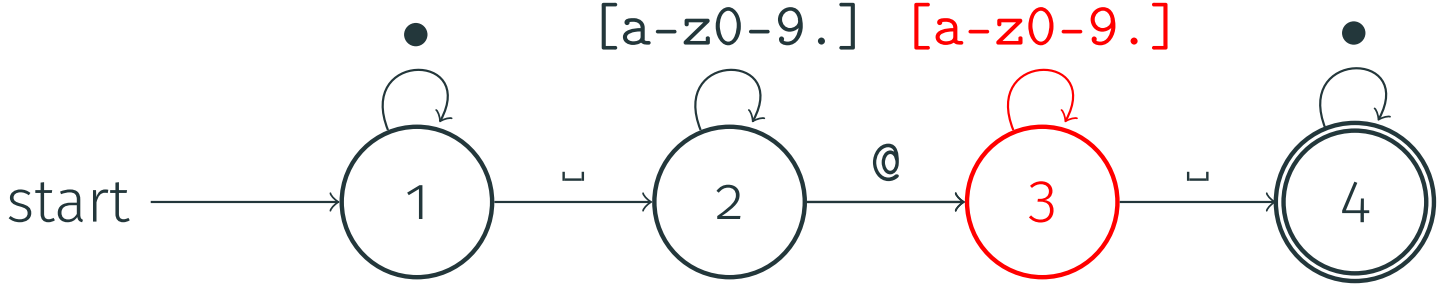

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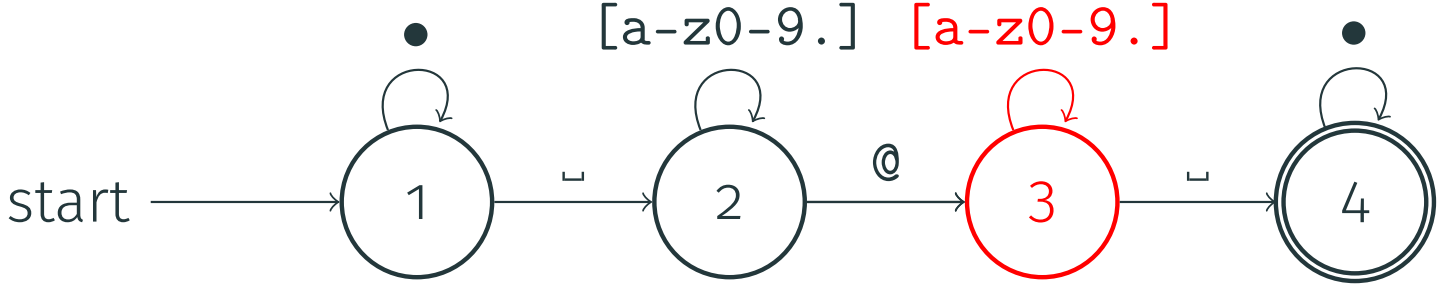

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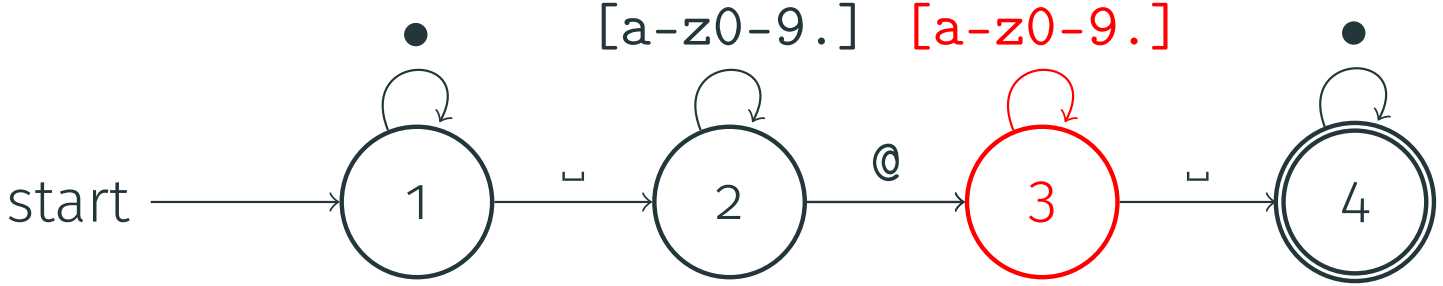

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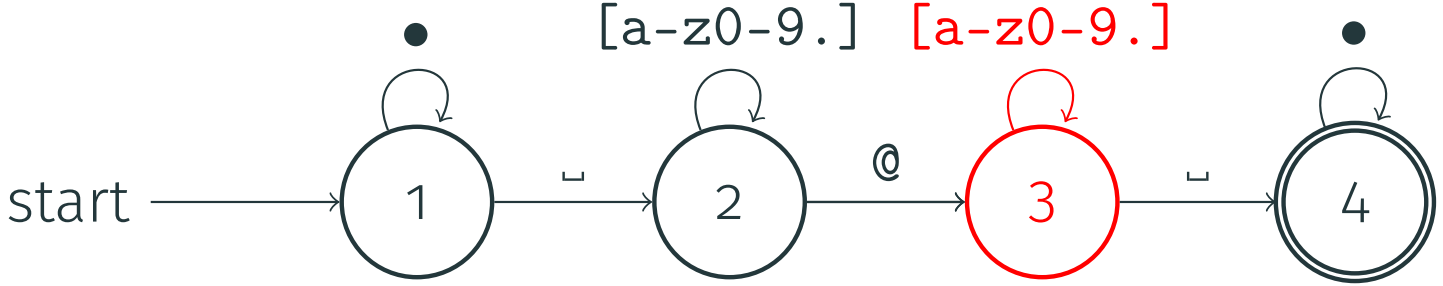

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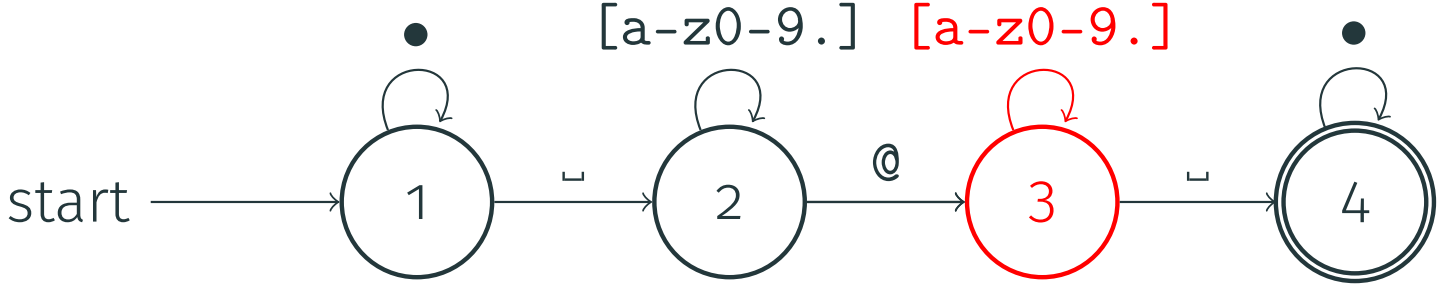

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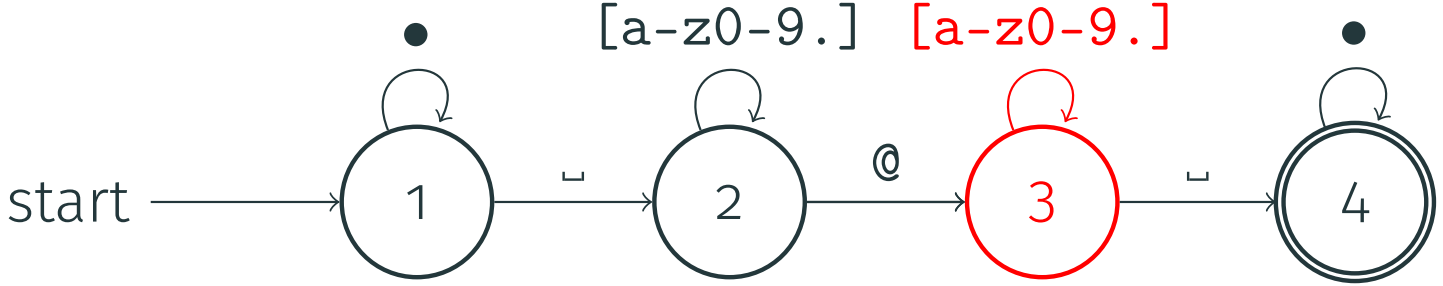

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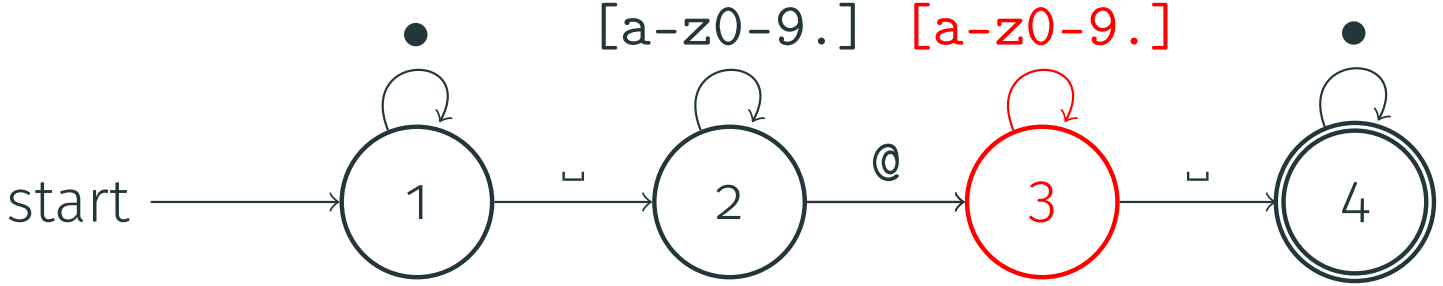

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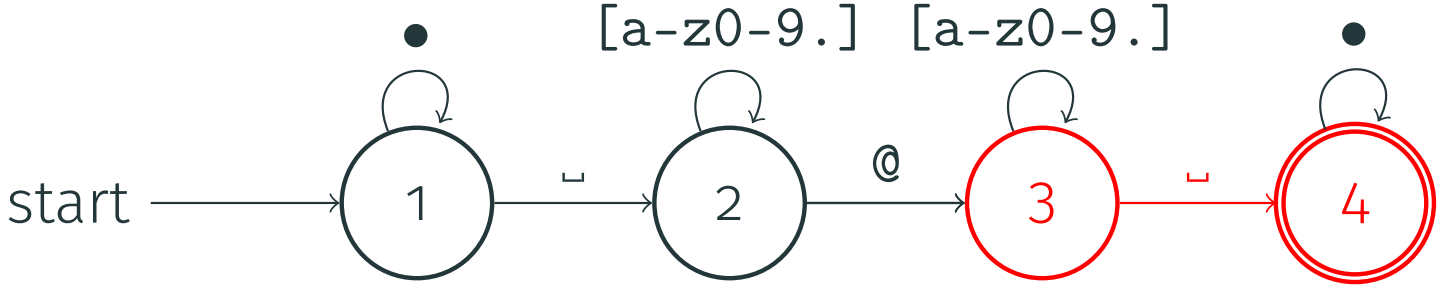

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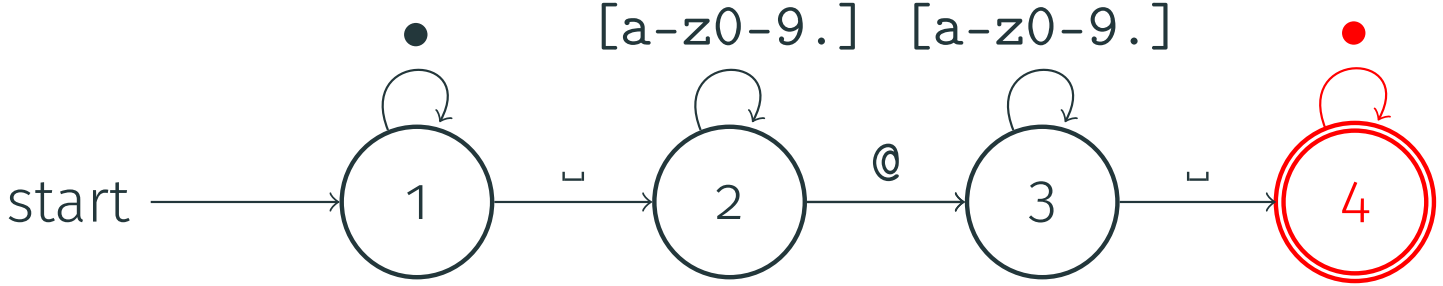

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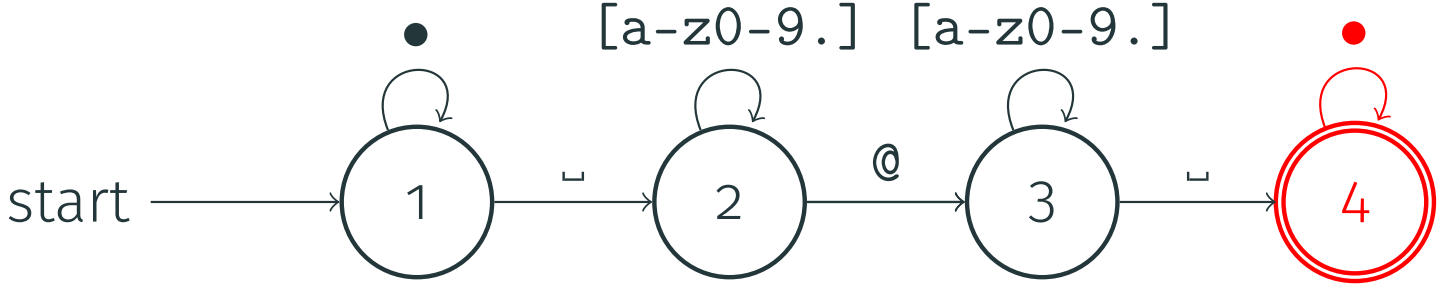

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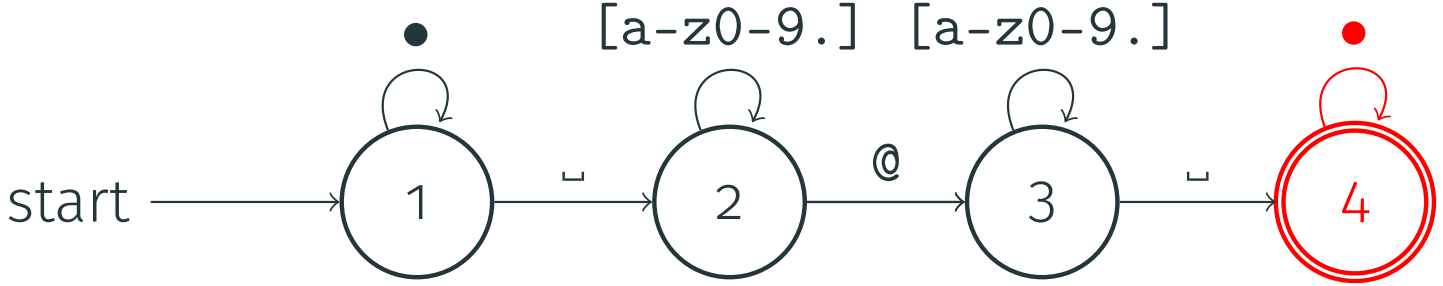

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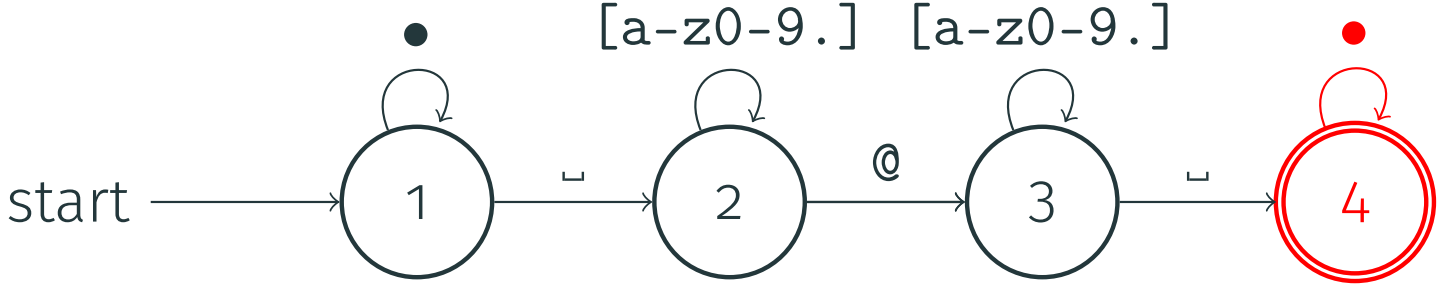

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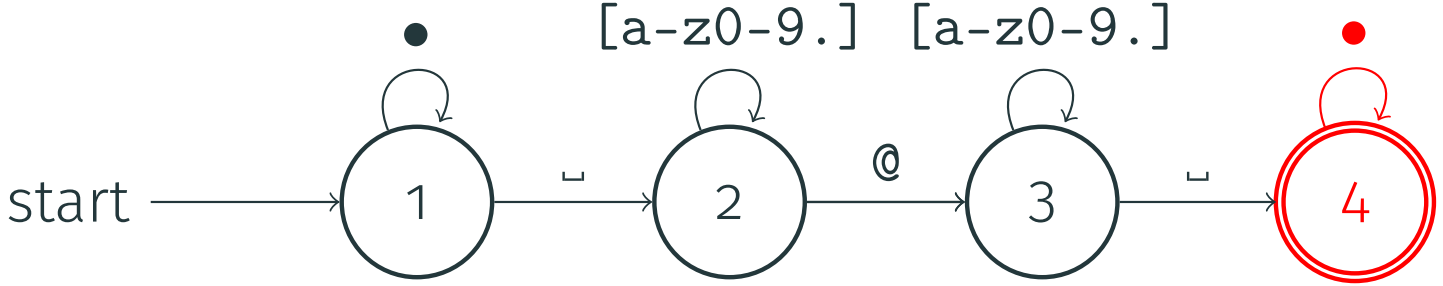

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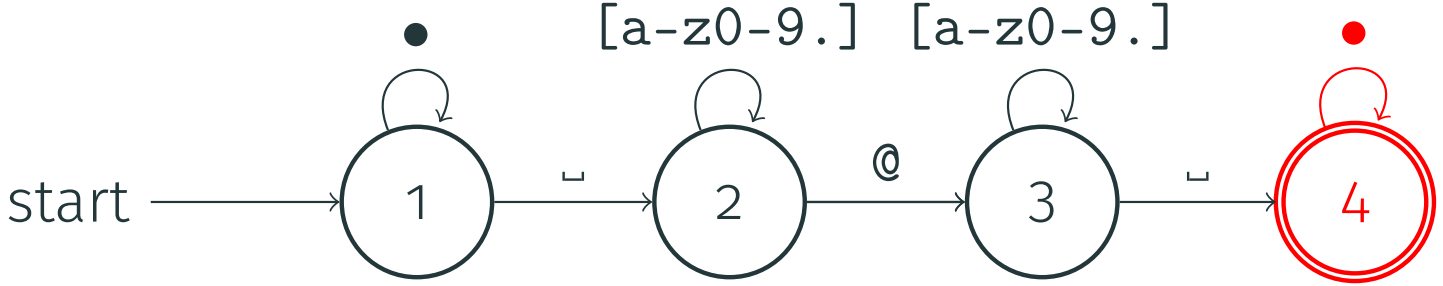

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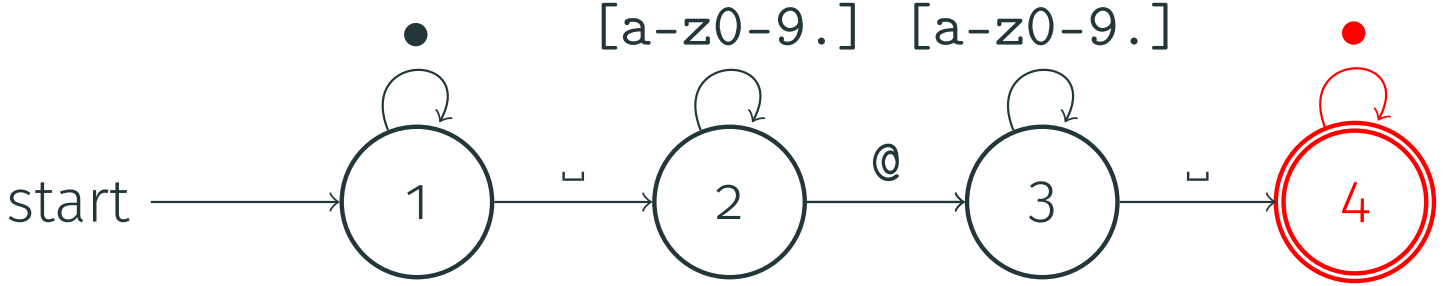

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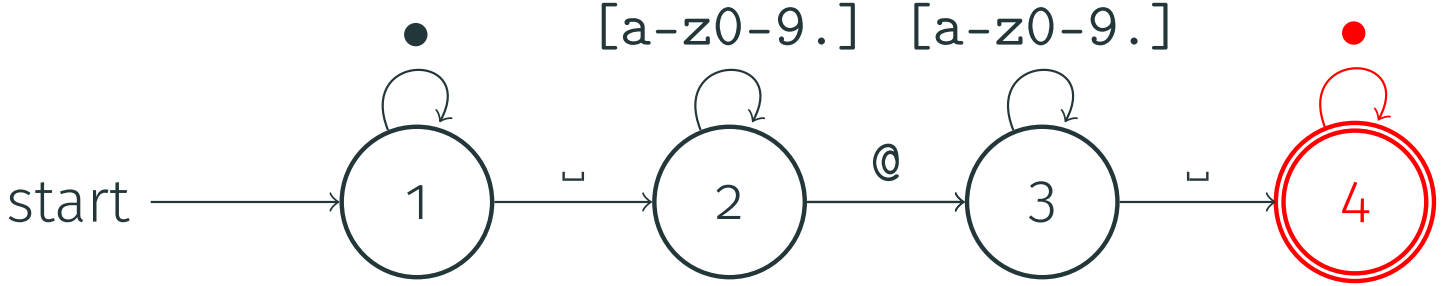

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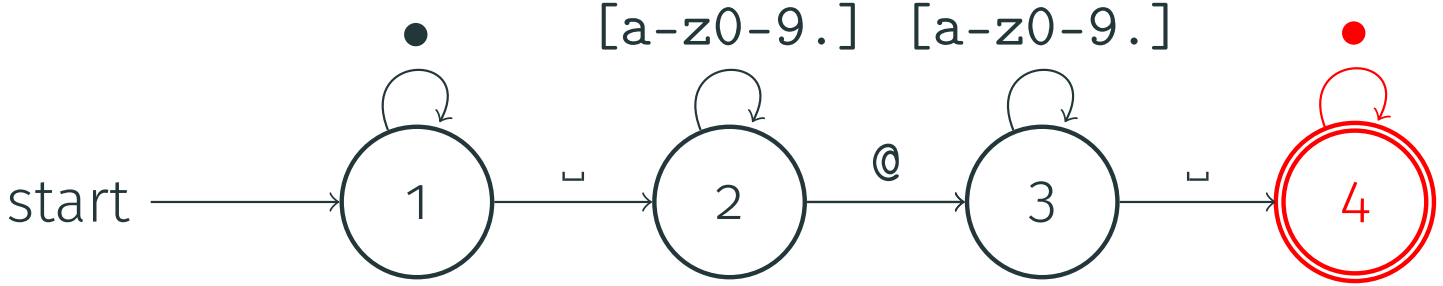

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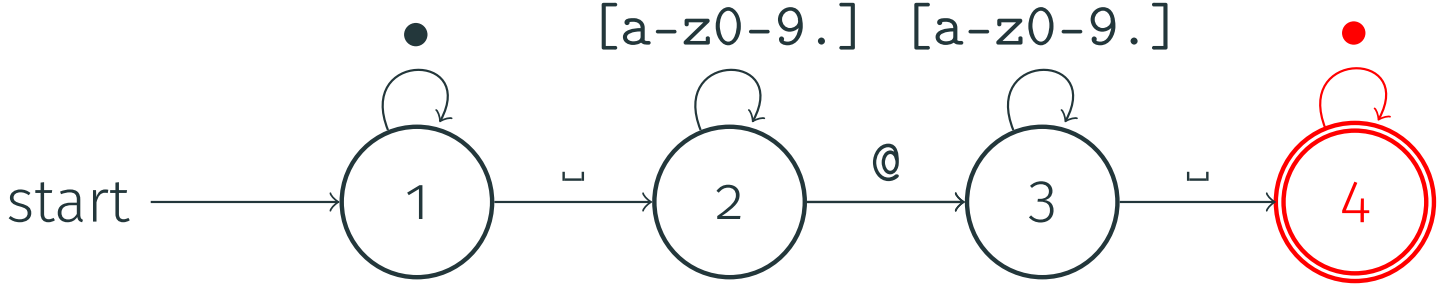

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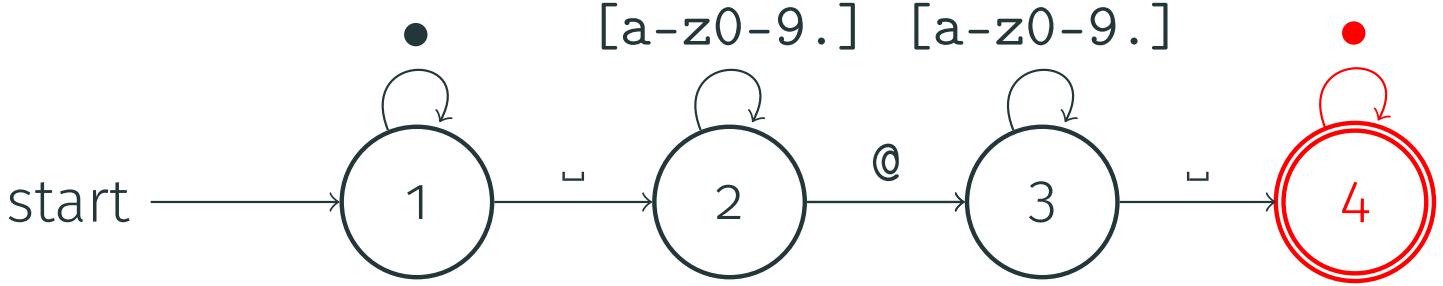

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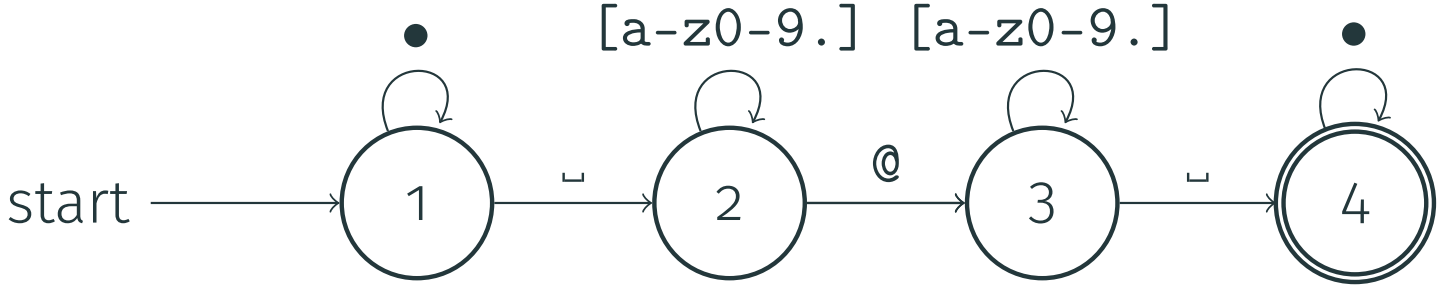

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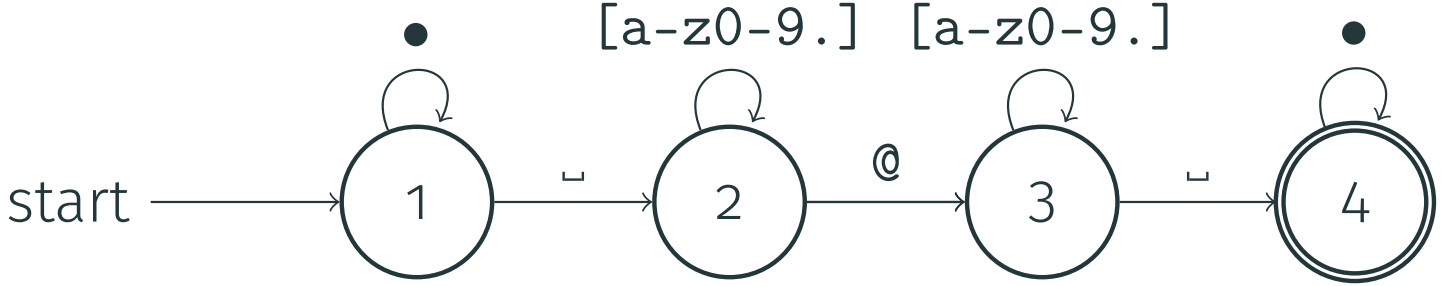

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- This only tests if the pattern occurs in the text!

  $\rightarrow$ ‘‘YES’’

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Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

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$\rightarrow$ We need a **different way** to measure complexity

Enumeration Algorithms

Idea: In real life, we do not want to compute all the matches
we just need to be able to enumerate matches quickly

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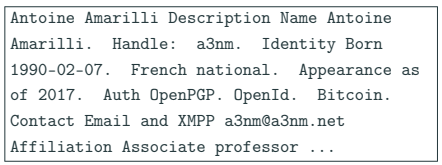
→ Formalization: enumeration algorithms

Formalizing Enumeration Algorithms

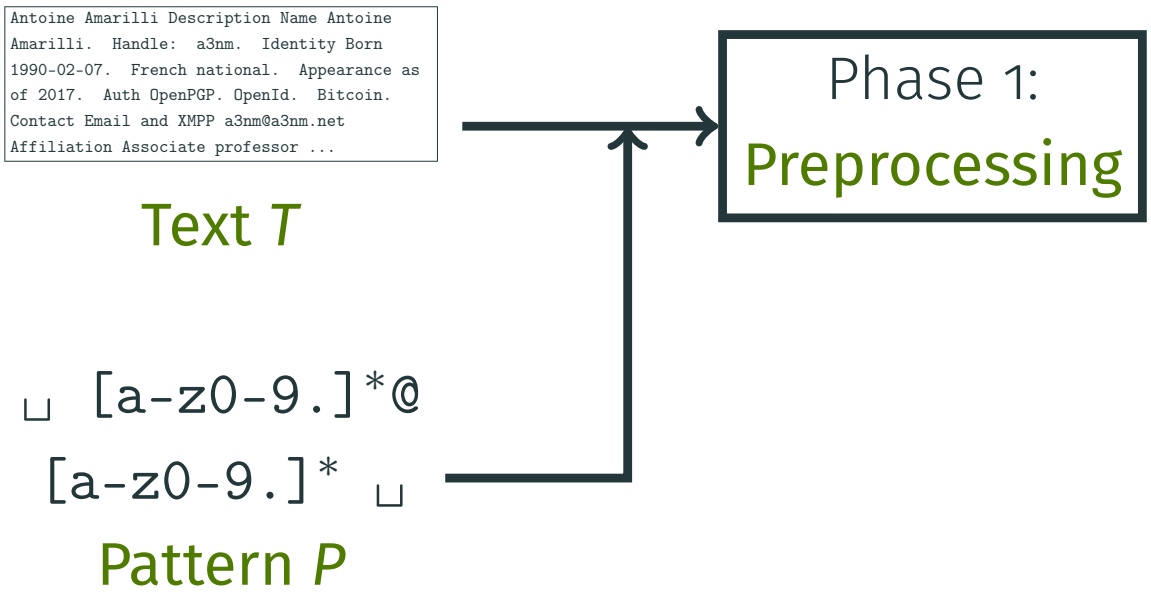


Text T

$\sqcup$ [a-z0-9.]*@ [a-z0-9.]* $\sqcup$

Pattern P

Formalizing Enumeration Algorithms



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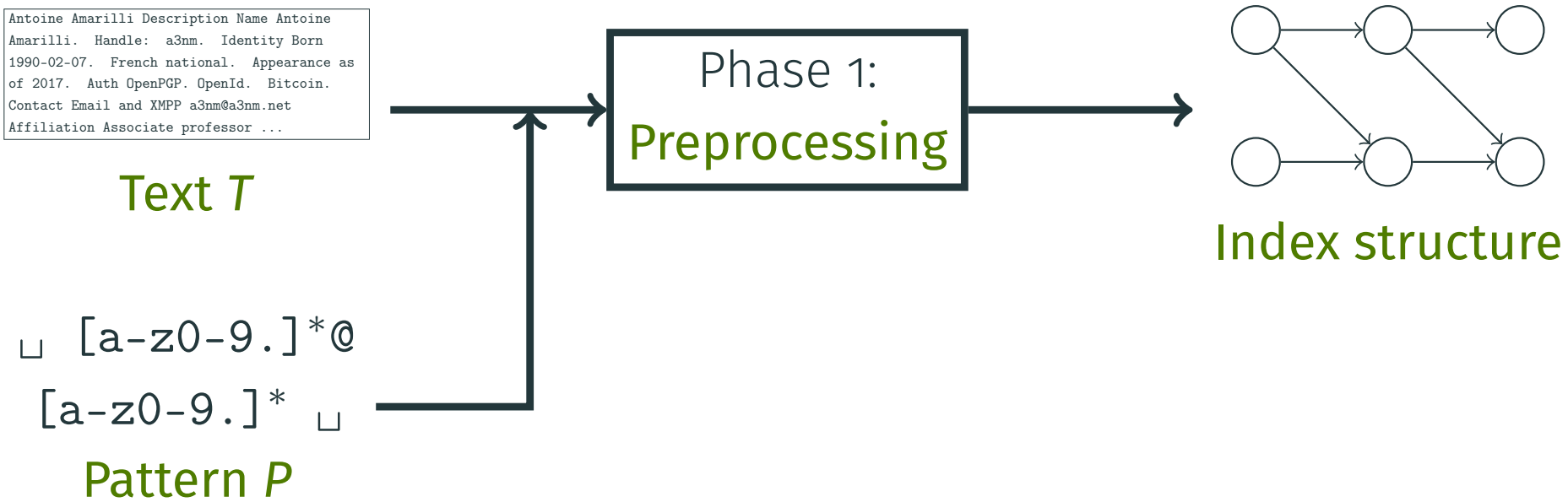

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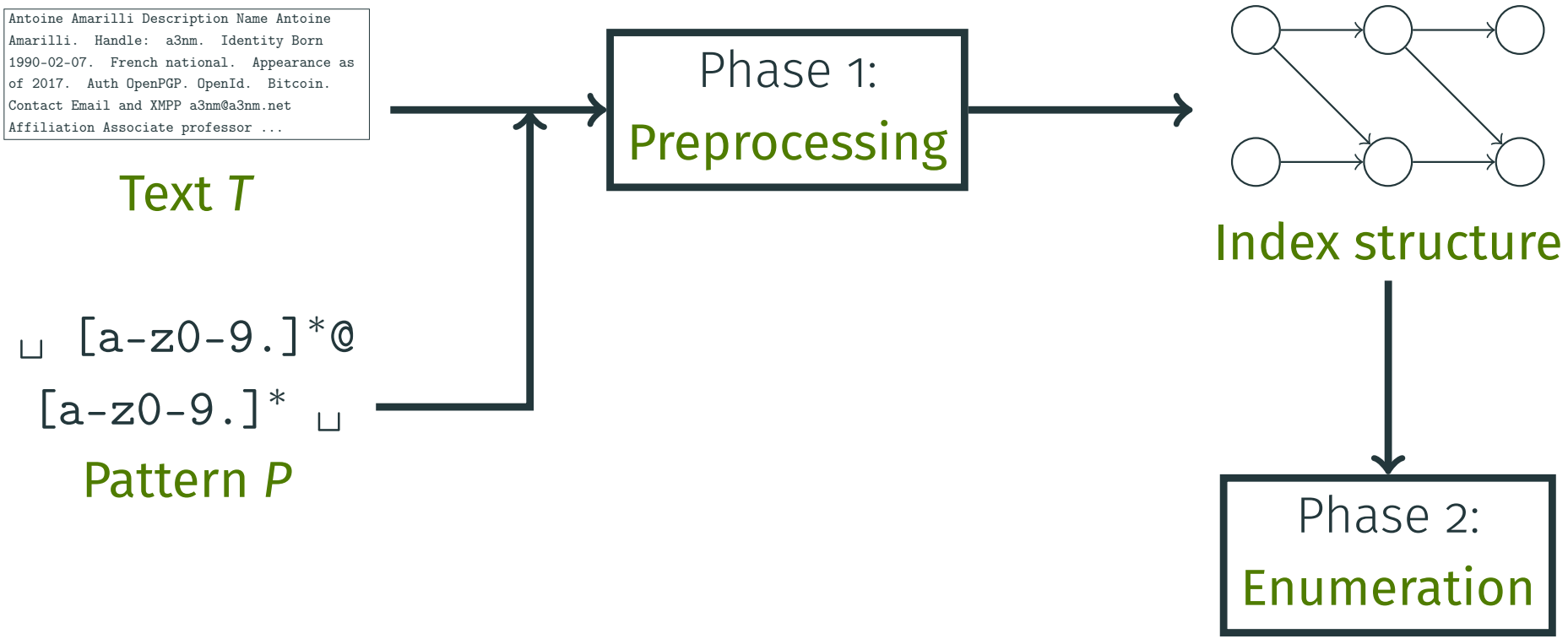

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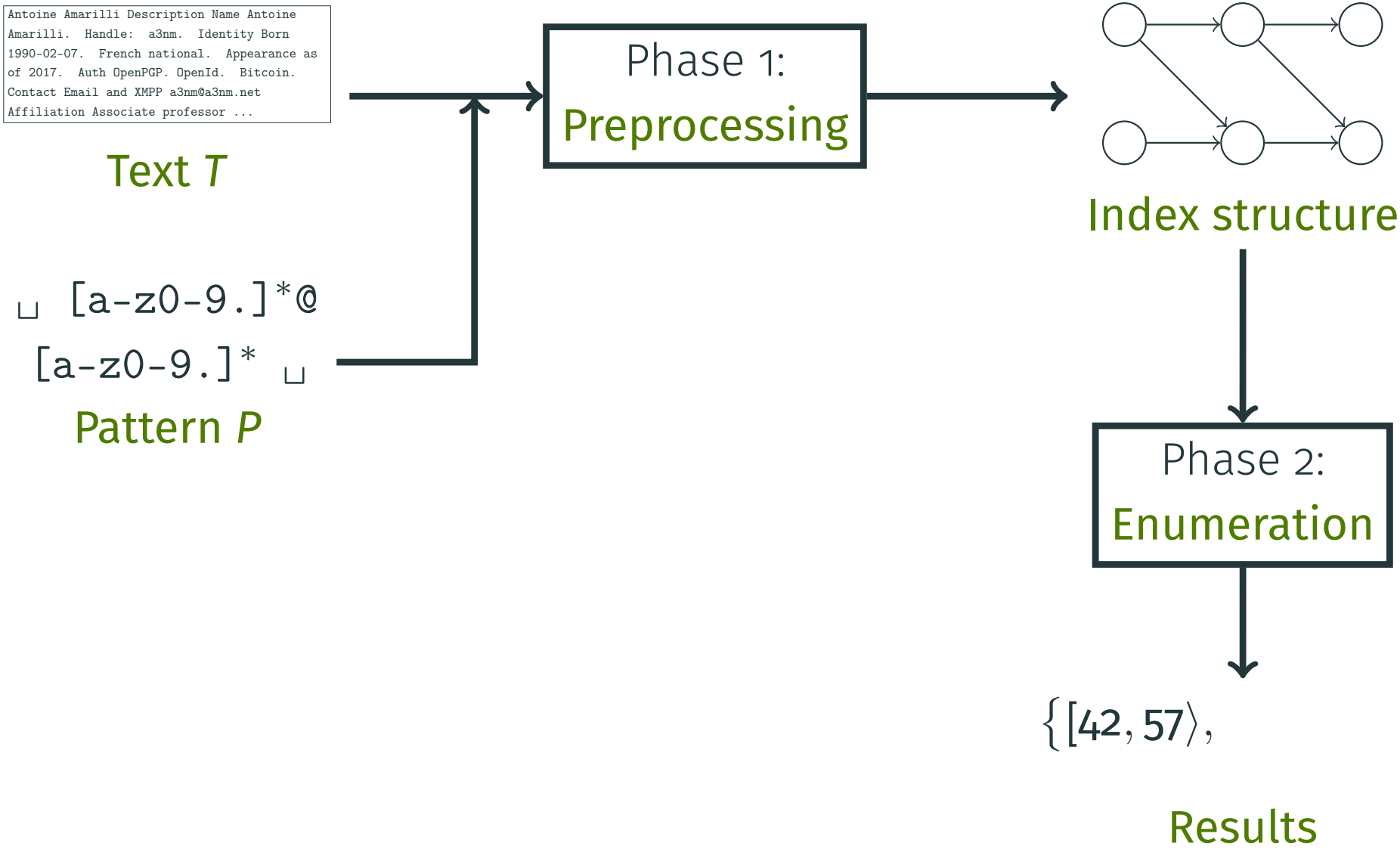

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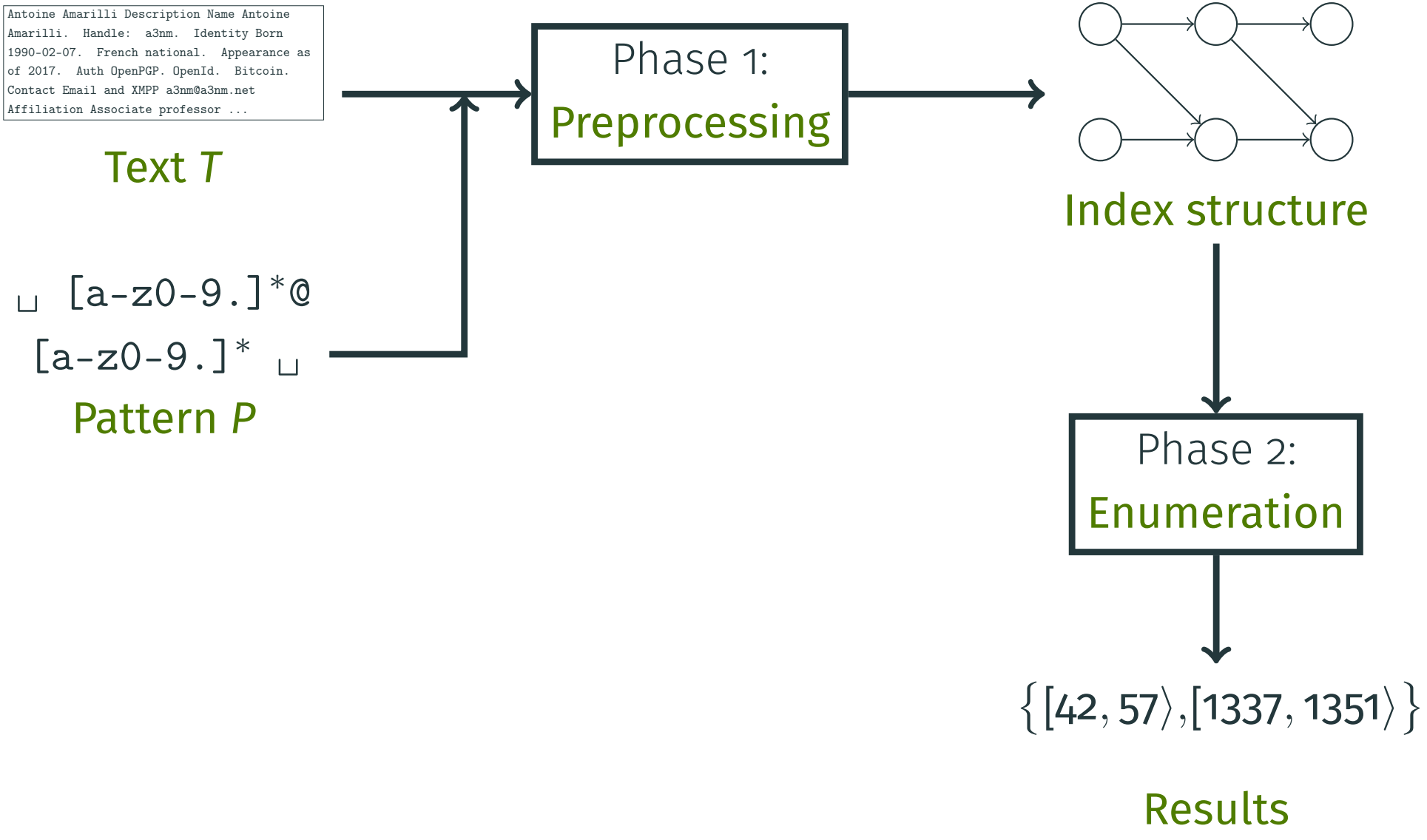

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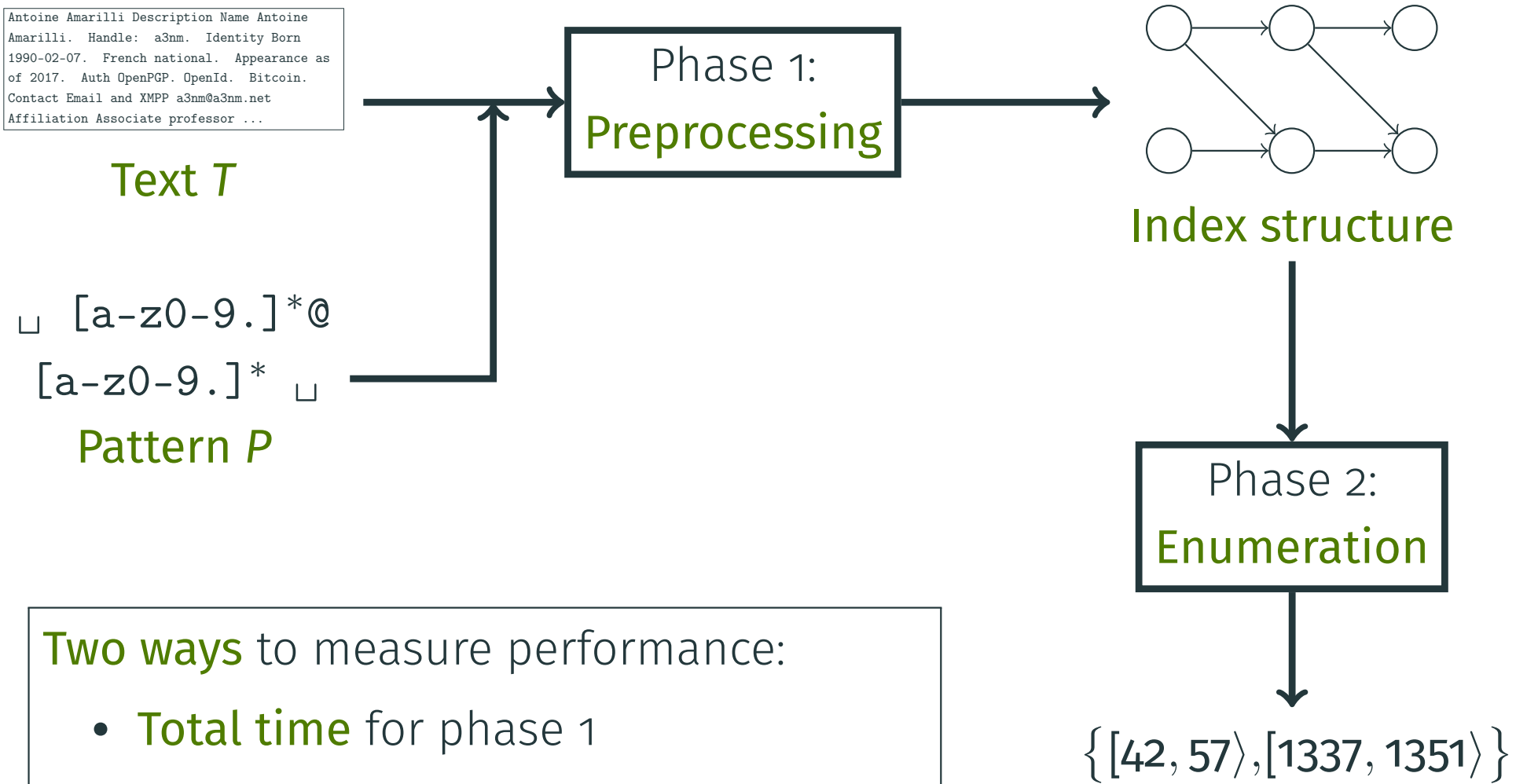

Two ways to measure performance:

- Total time for phase 1
- Delay between two results in phase 2

... as a function of the text and pattern

Complexity of Enumeration Algorithms

- Recall the inputs to our problem:
  - A text T

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- What is the **delay** of the **naive algorithm**?

$\rightarrow$ it is the **maximal time** to find the next **matching substring**

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Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France. Studies PhD in computer science
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  - A **pattern** P given as a regular expression

$$P := \sqcup \texttt{[a-z0-9.]}^* \texttt{@} \texttt{[a-z0-9.]}^* \sqcup$$

- What is the **delay** of the **naive algorithm**?

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  $\rightarrow$ i.e. $O(|T|^2 \times |A|)$, e.g., if only the **beginning** and **end** match

Complexity of Enumeration Algorithms

- Recall the **inputs** to our problem:
  - A **text** T

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$\rightarrow$ Can we do **better**?

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Theorem

We can enumerate all matches of a pattern P on a text T with:

- *Preprocessing in $O(|T| \times Poly(P))$*
- *Delay polynomial in P and independent from T*

Automaton Formalism

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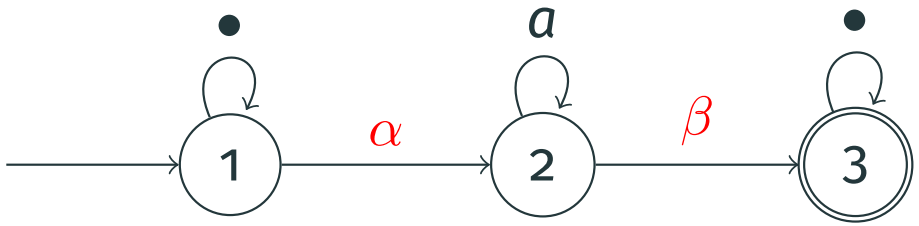

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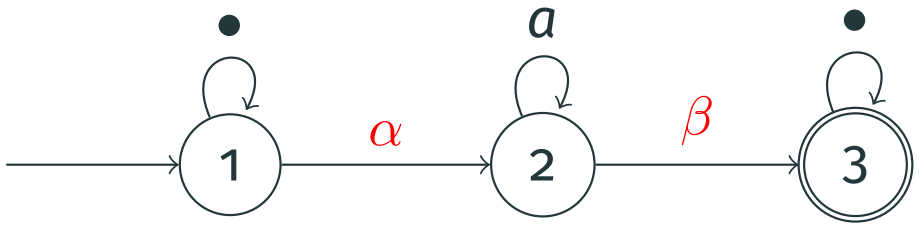

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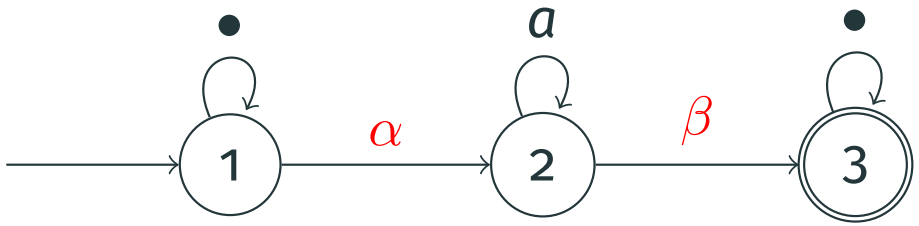

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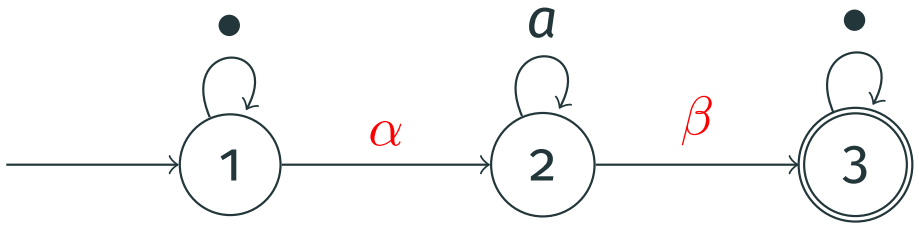

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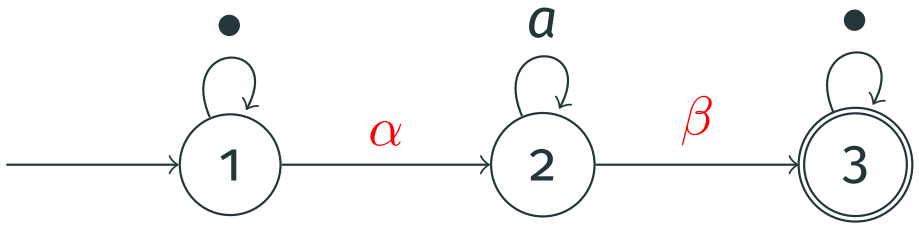

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    A has an accepting run reading α at position i and β at j
- Assumption: There is no run for which A reads the same capture variable twice at the same position
- Challenge: Because of nondeterminism we can have many different runs of A producing the same tuple!

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Compute a product DAG of the text T and of the automaton A

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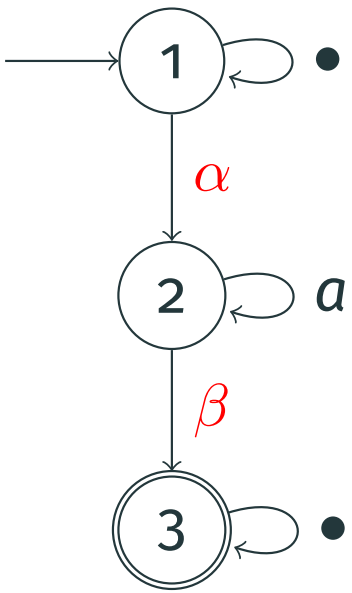

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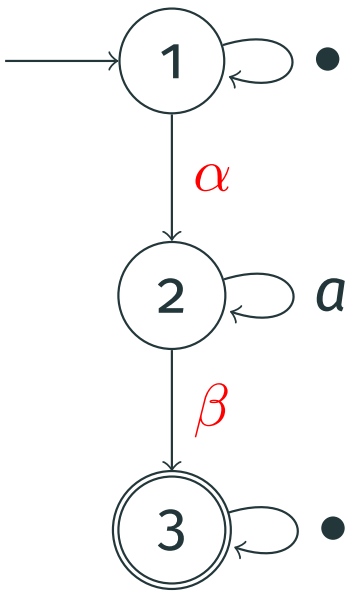

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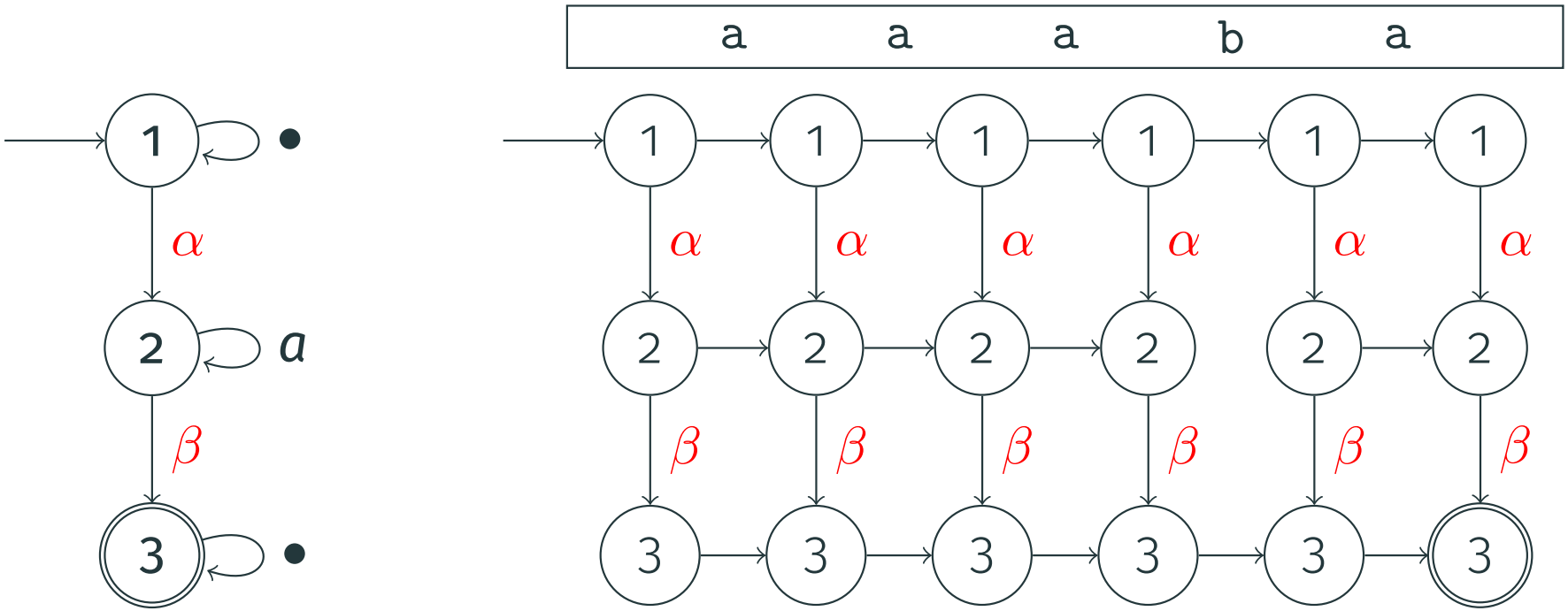

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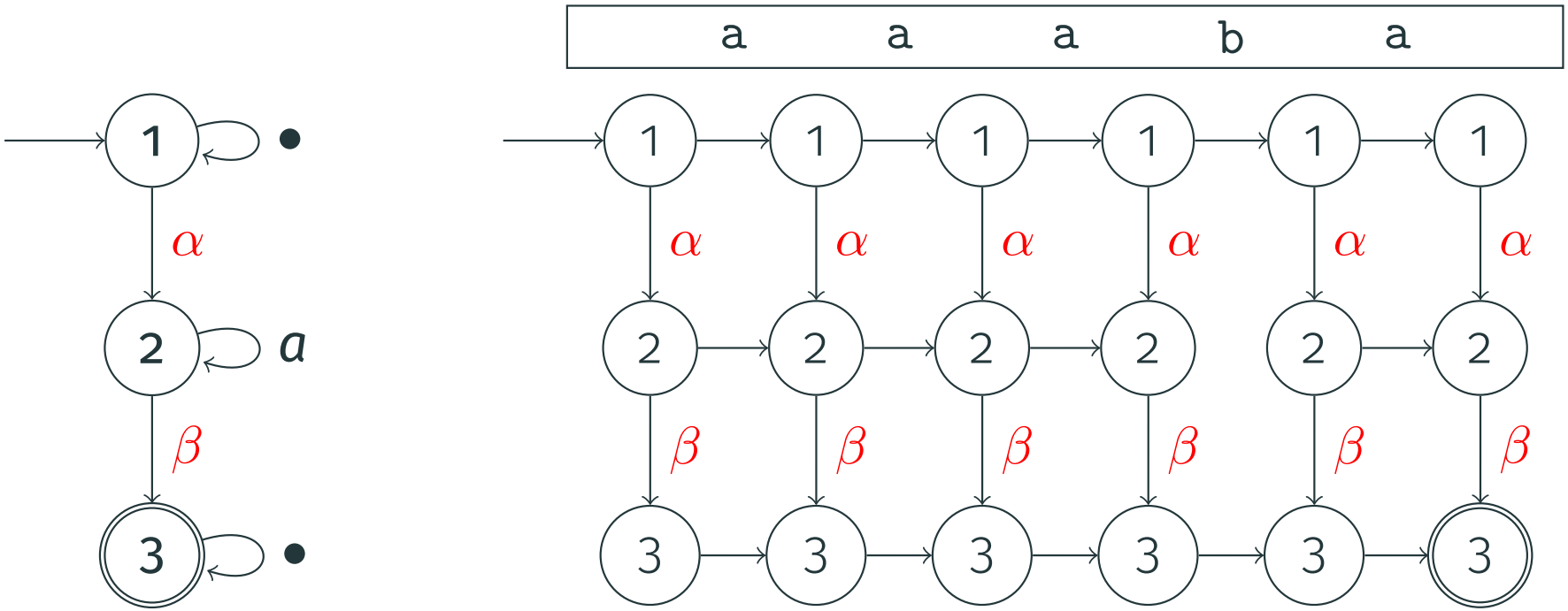

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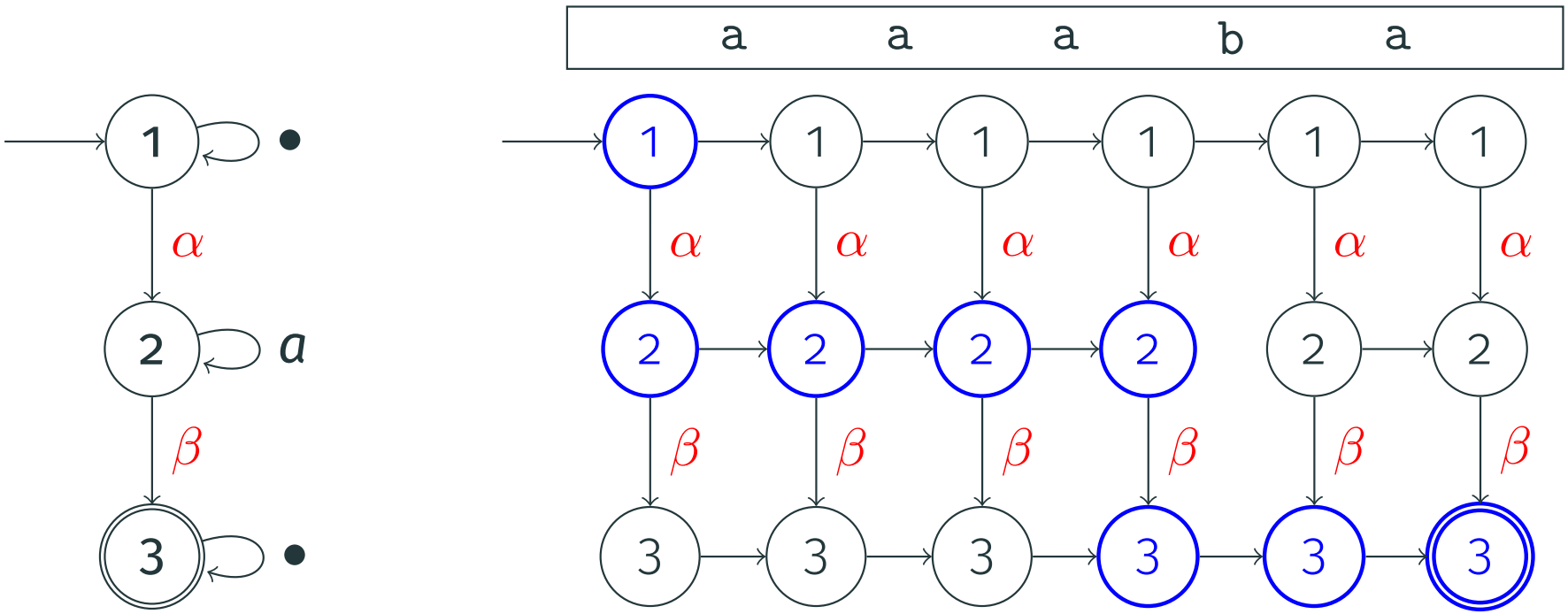

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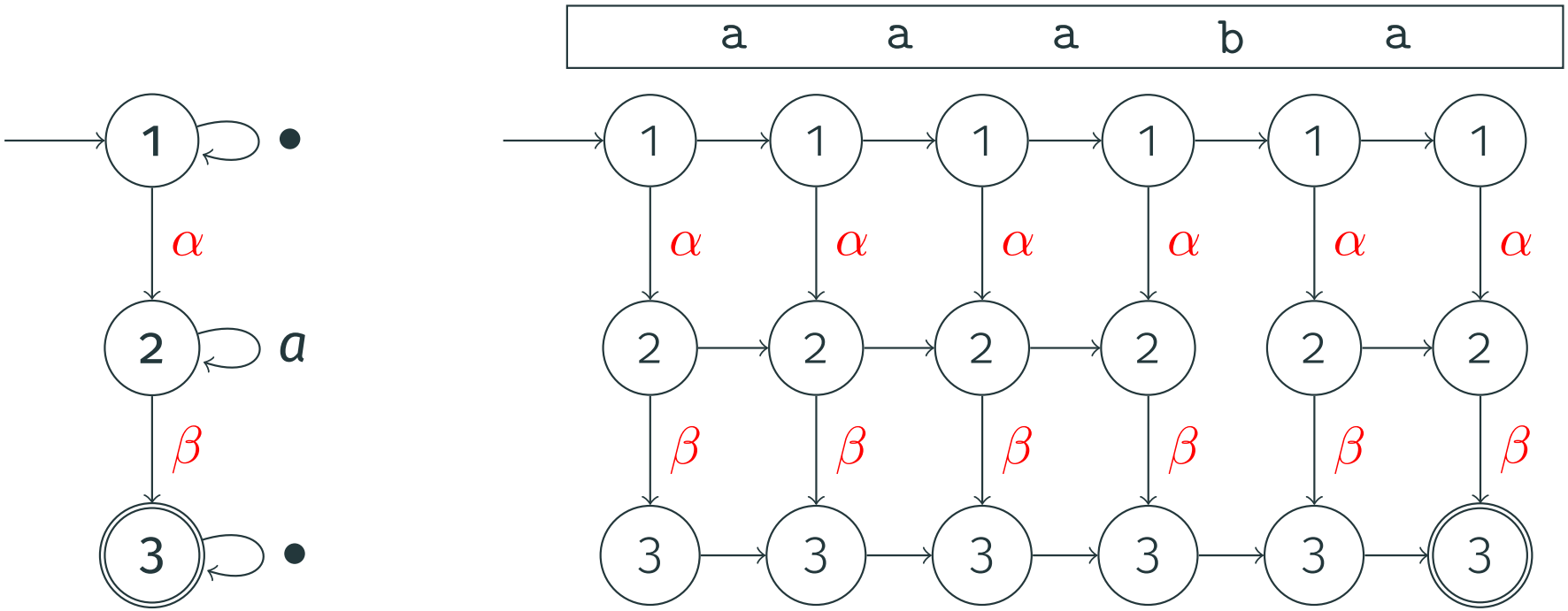

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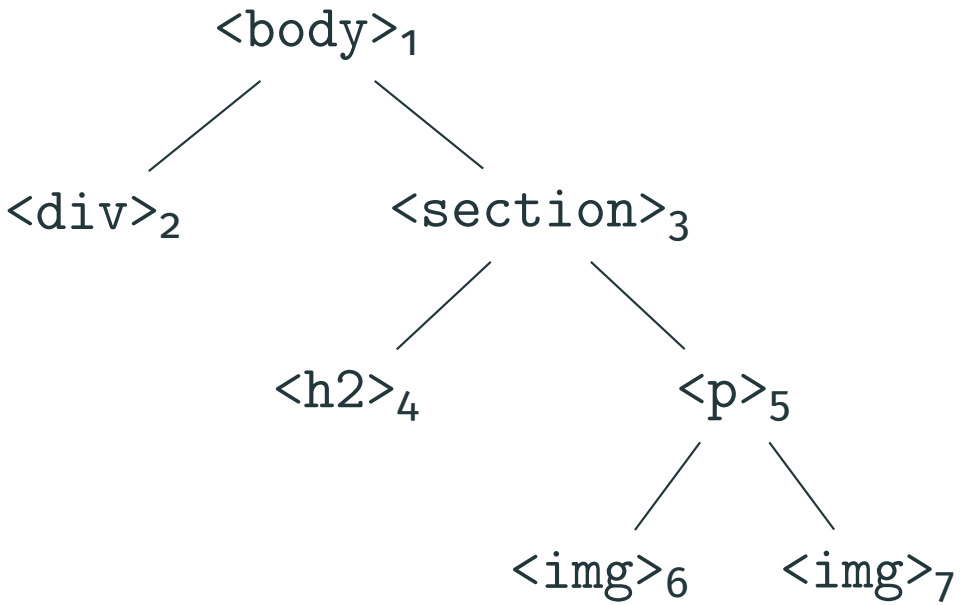
→ **Challenge:** Enumerate paths but avoid **duplicate matches** and do not **waste time** to ensure constant delay

Extension: From Text to Trees

Pattern Matching on Trees

- The data T is no longer text but is now a tree:



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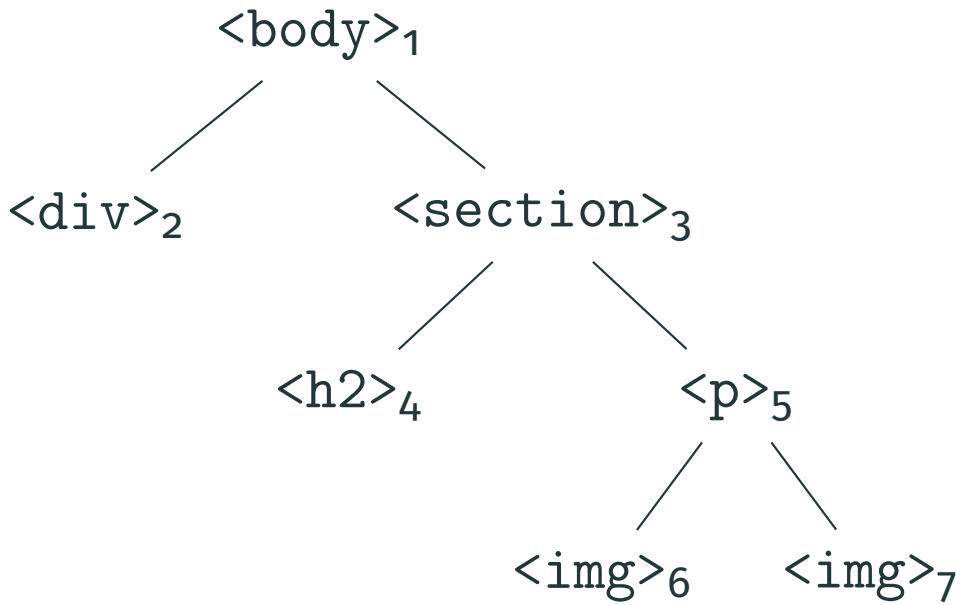

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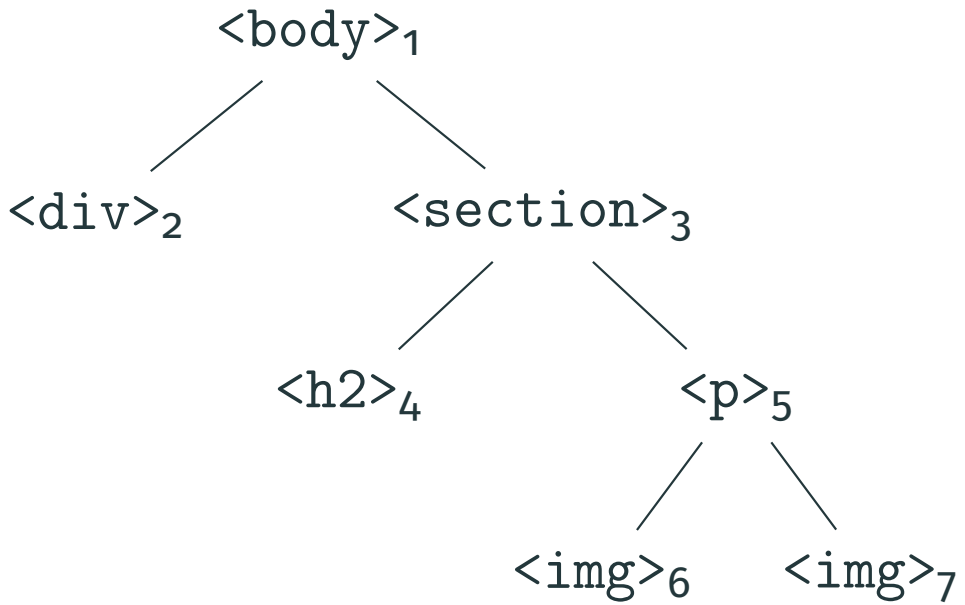

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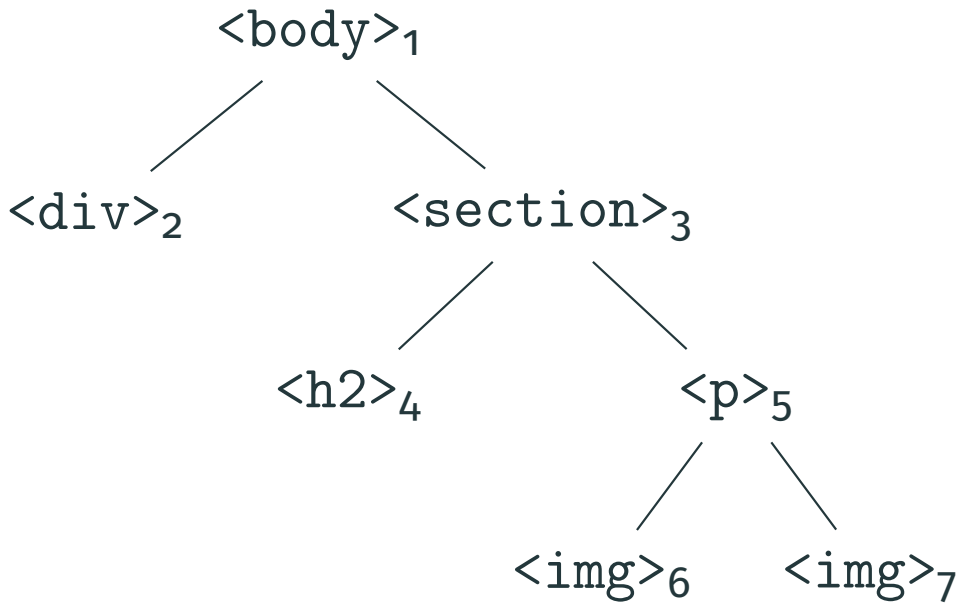

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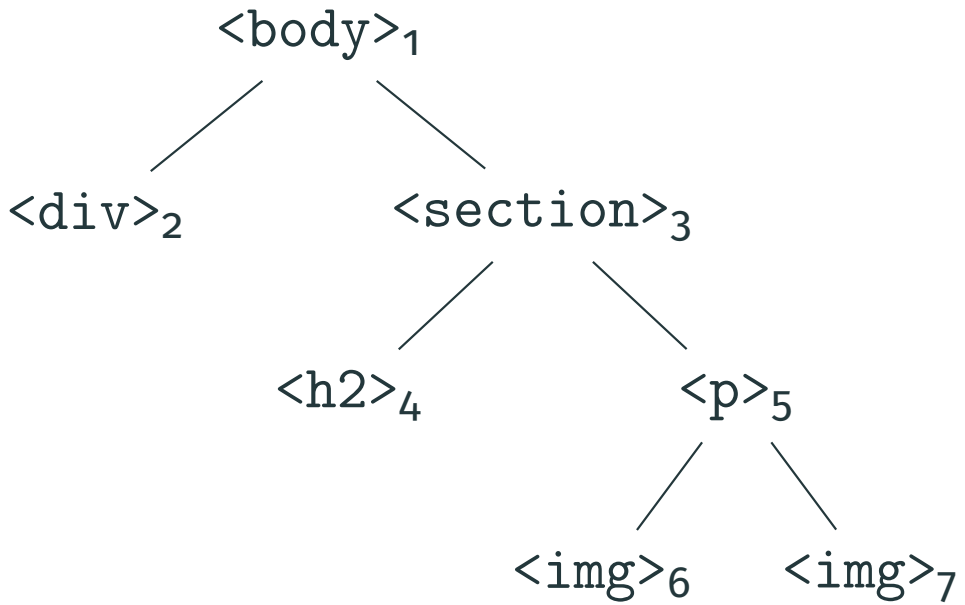

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- Results: $\langle \alpha : 4, \beta : 6 \rangle$, $\langle \alpha : 4, \beta : 7 \rangle$

Definitions and Results on Trees

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$\rightarrow$ We are working on proving the following:

Conjecture

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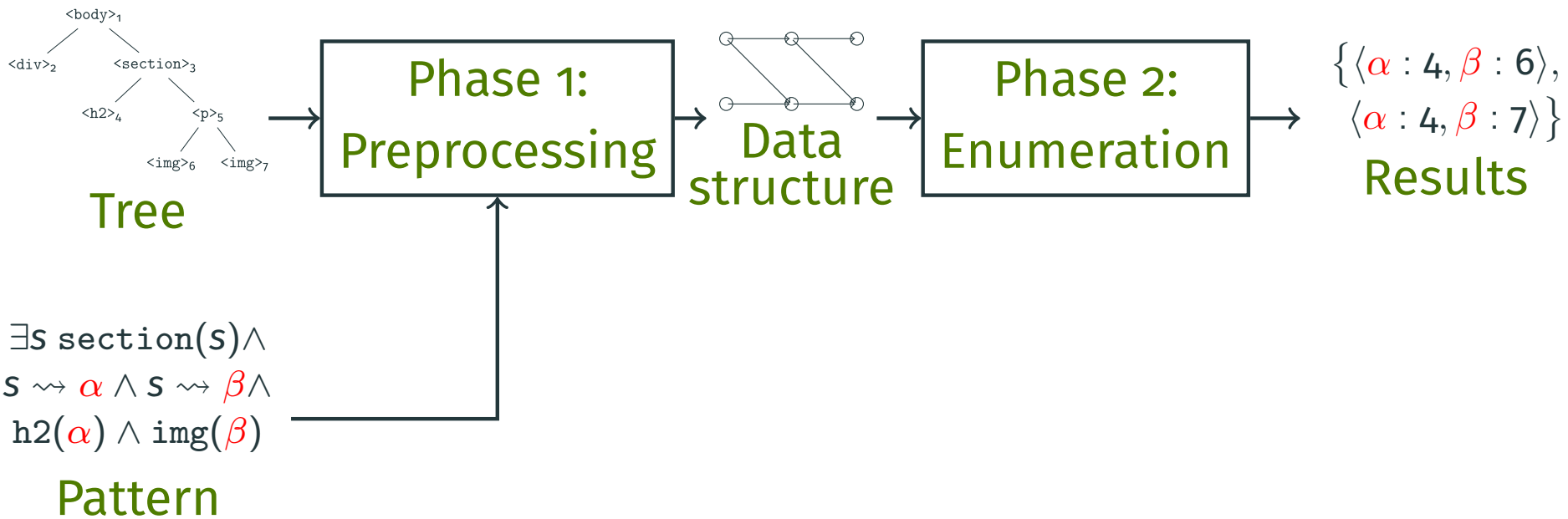
Proof Idea for Trees: Structure

Similar structure to the previous proof, but with a circuit:



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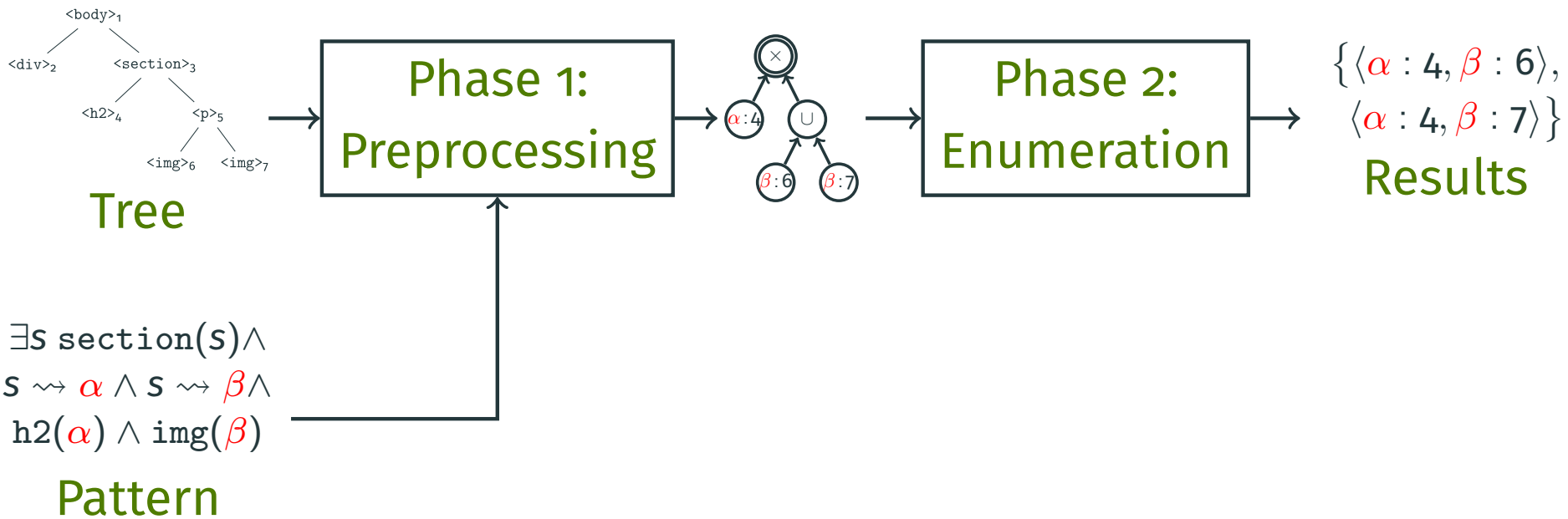
- Preprocessing: Compute a circuit representation of the answers
- Enumeration: Apply a generic algorithm on the circuit



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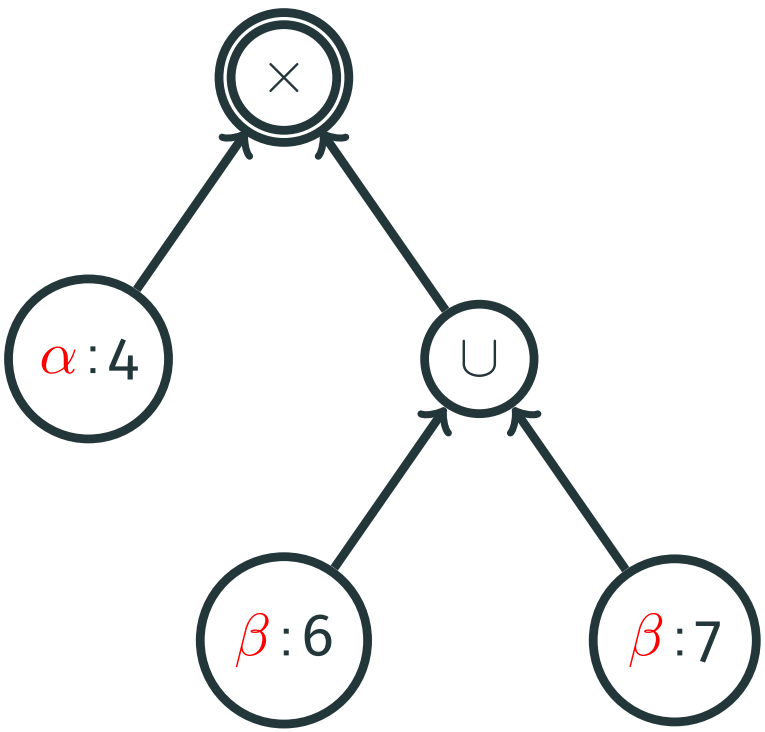

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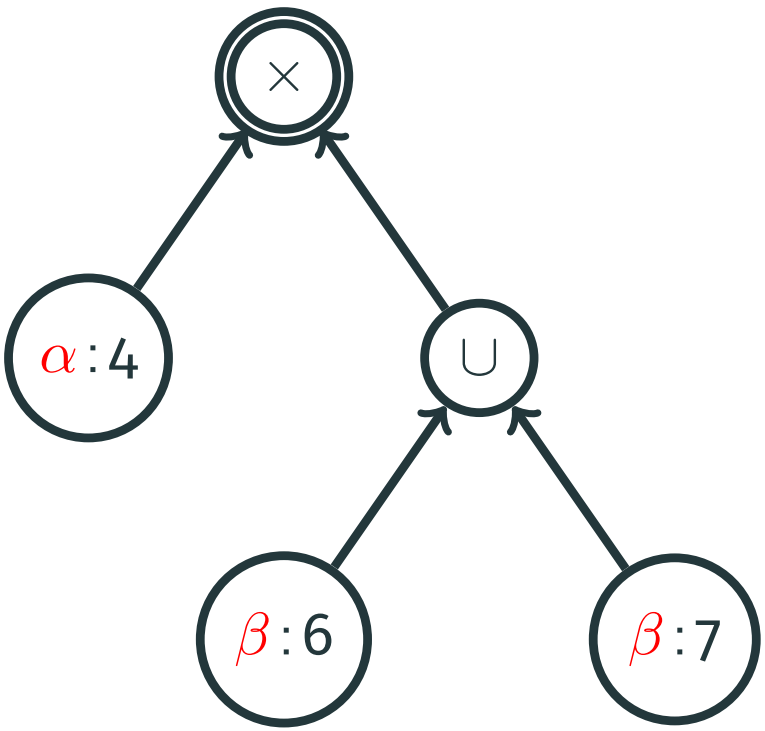

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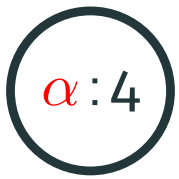
- **Variable gate** 

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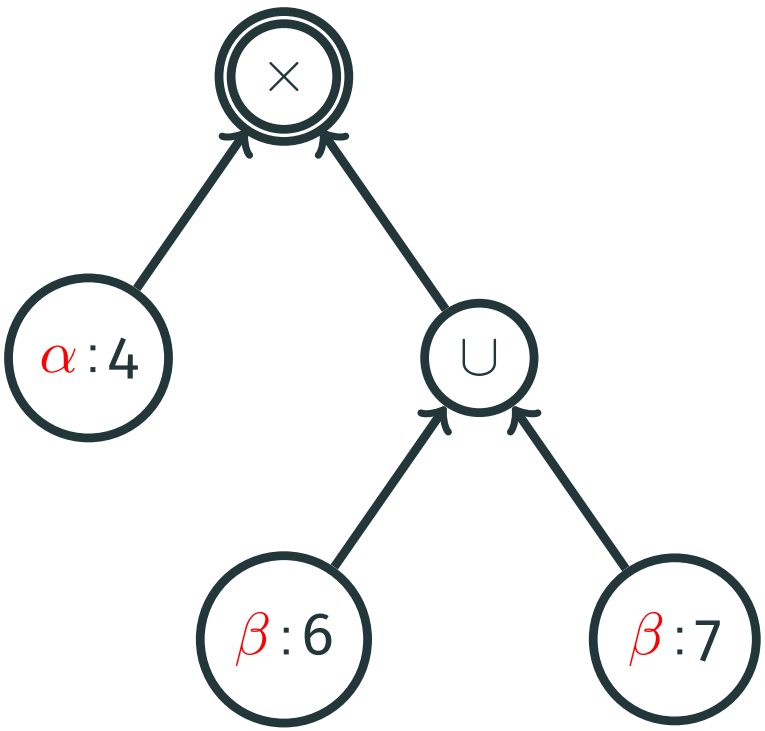

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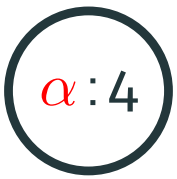
- **Variable gate** 

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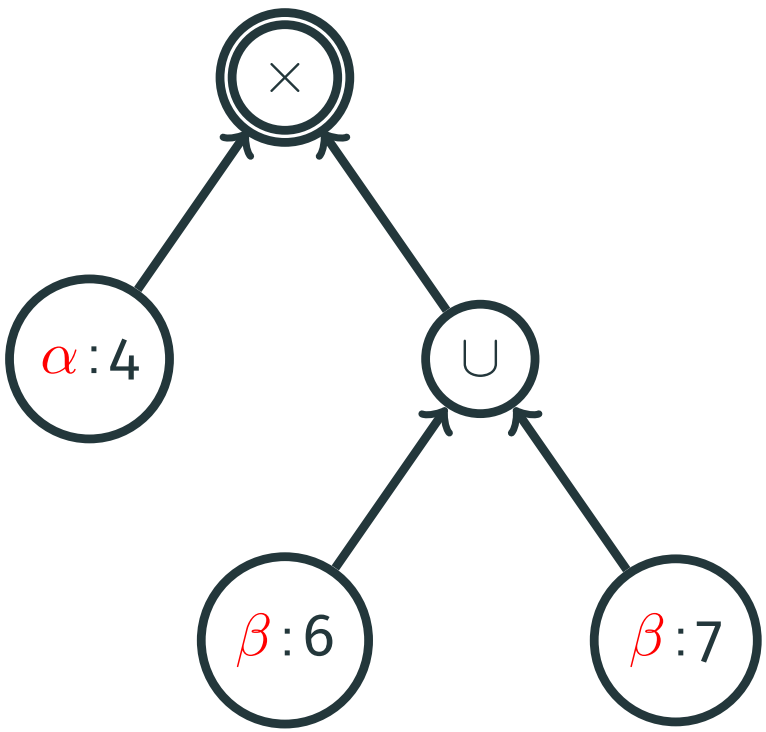

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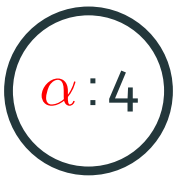
- **Variable gate**  :

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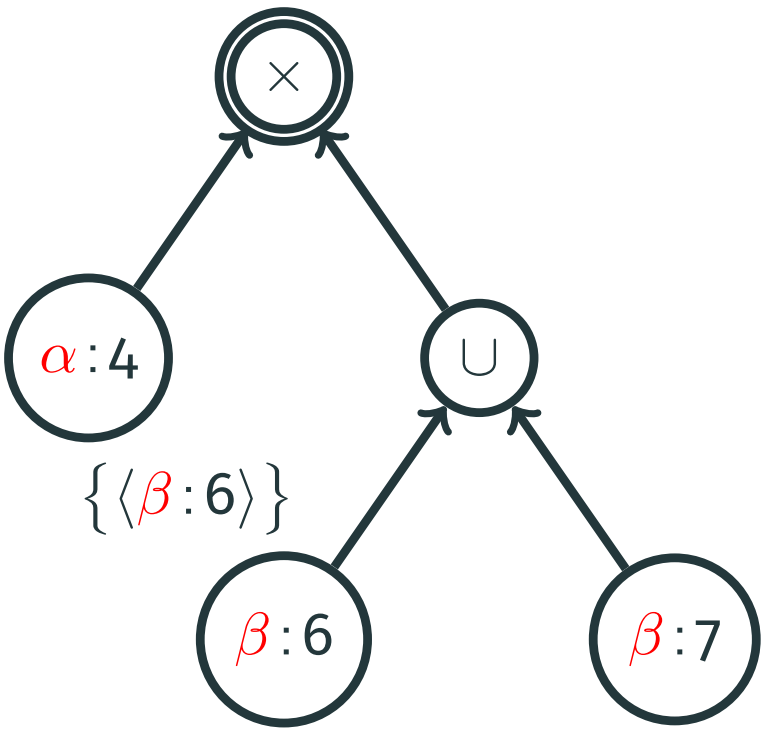

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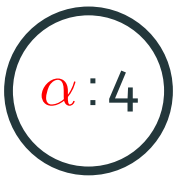
- **Variable gate**  :

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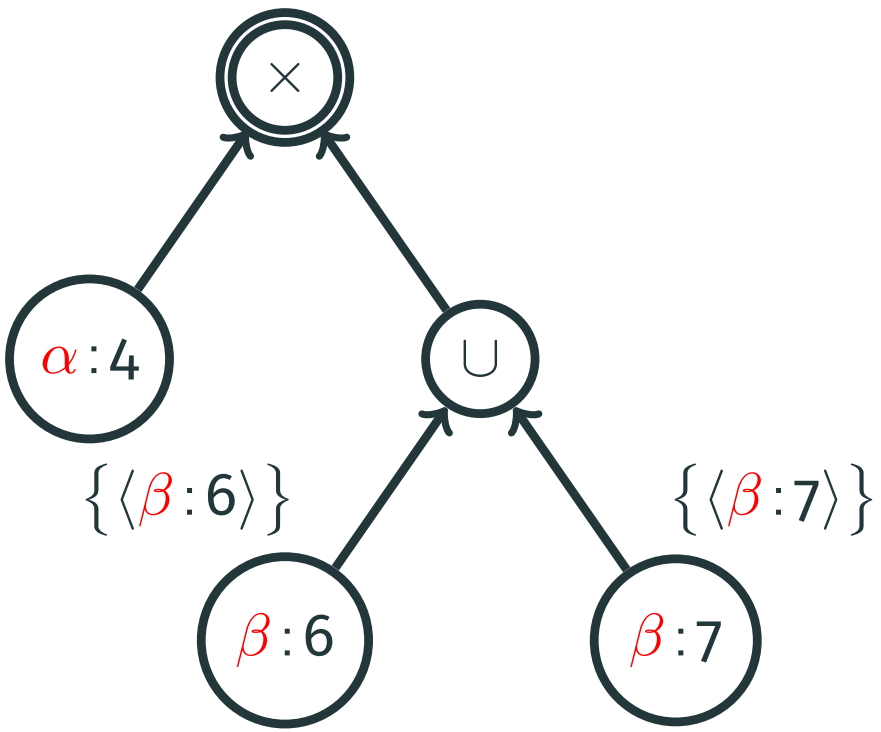

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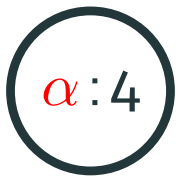
- **Variable gate**  :

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Proof Idea for Trees: Set Circuits

A **set circuit** represents a **set of answers** to a pattern $P(\alpha, \beta)$

- **Singleton** $\alpha : 6 \rightarrow$ *"the variable* α *is mapped to node* 6*"*
- **Tuple** $\langle \alpha : 4, \beta : 6 \rangle$: tuple of singletons
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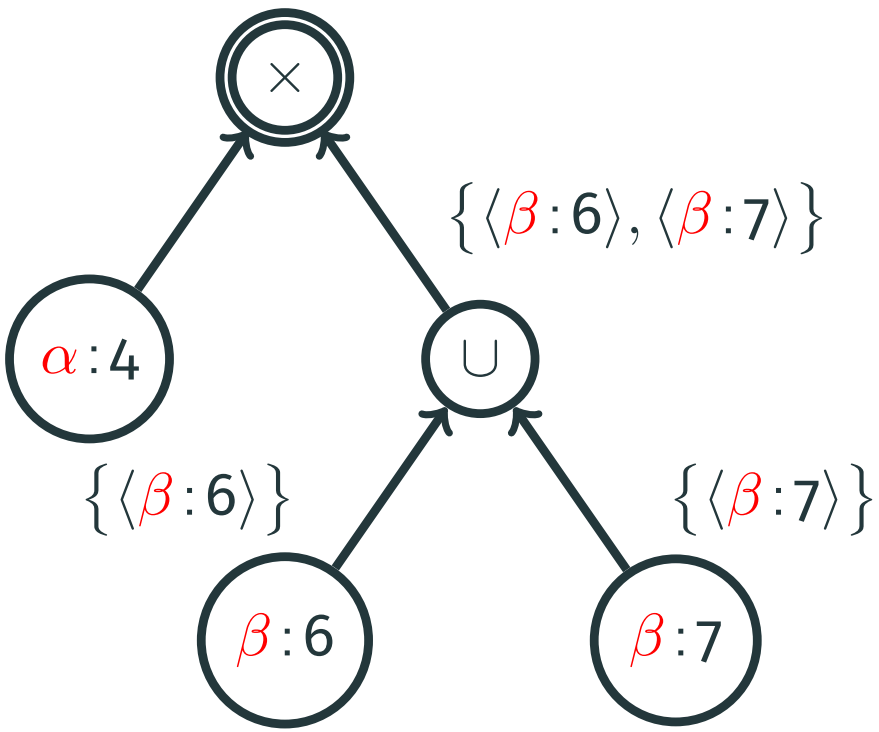

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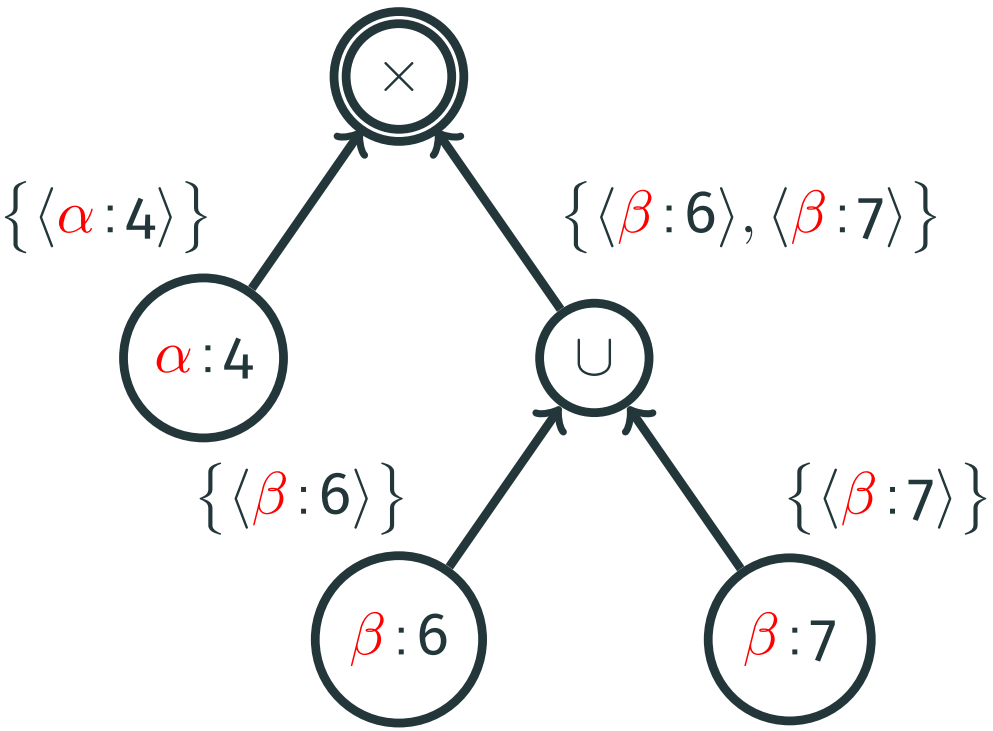

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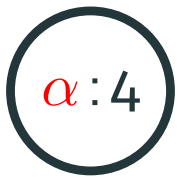
- **Variable gate** 

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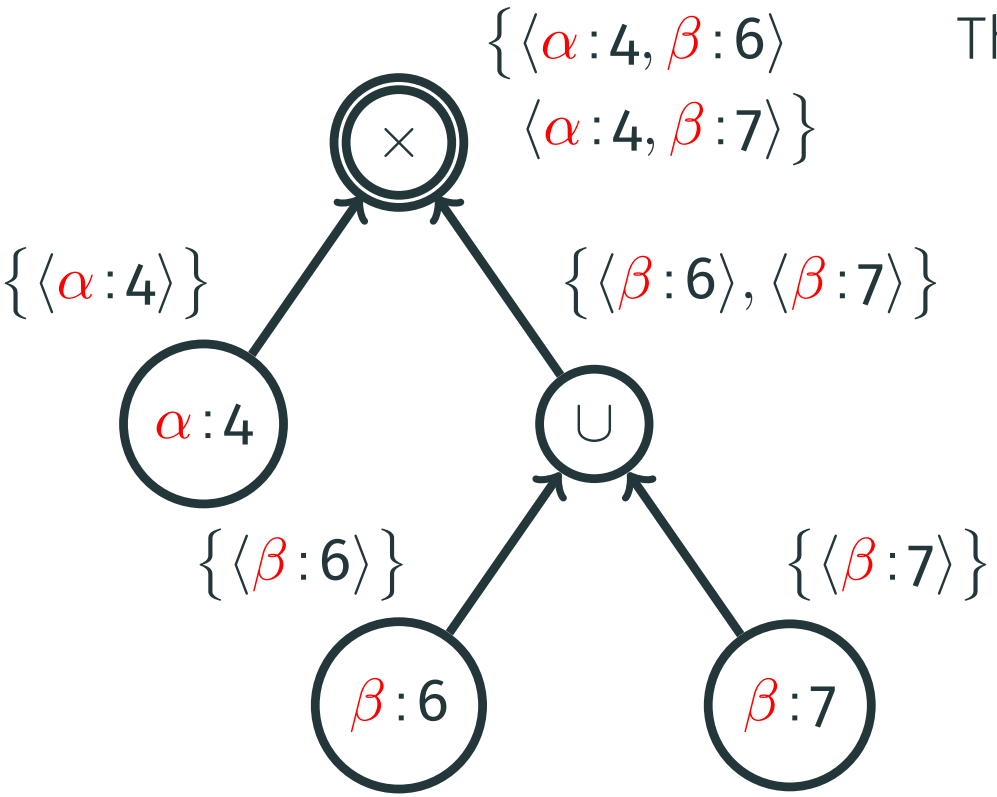

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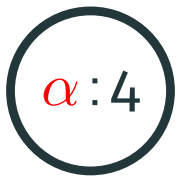
- **Variable gate**  :

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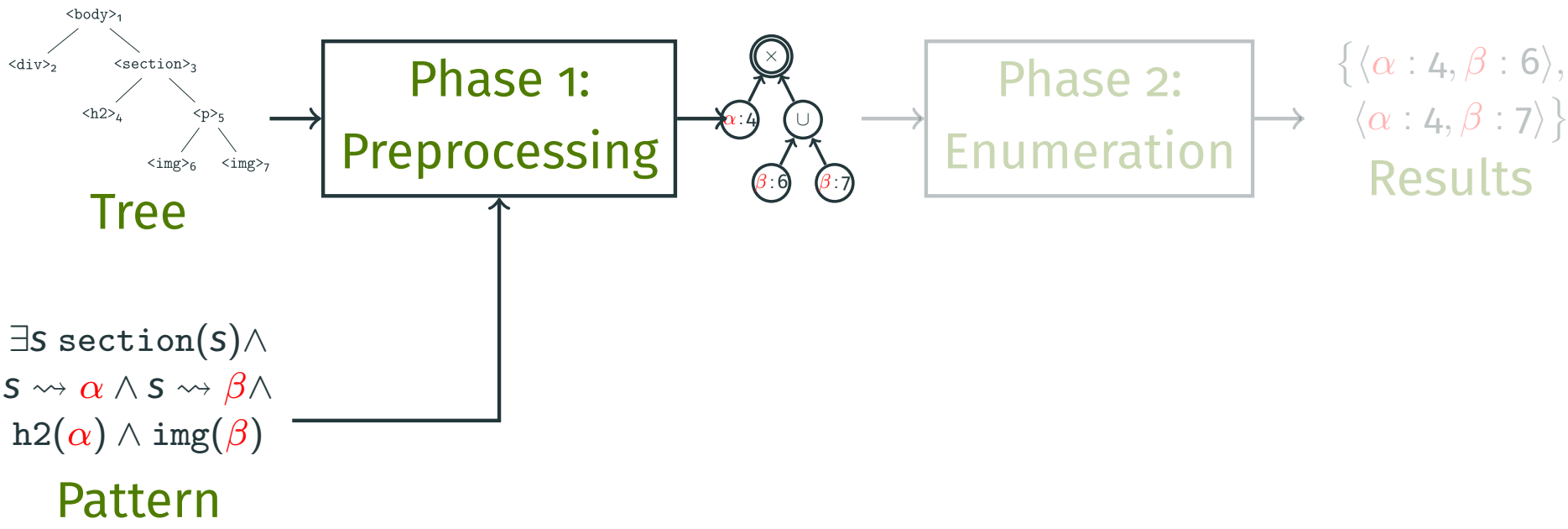
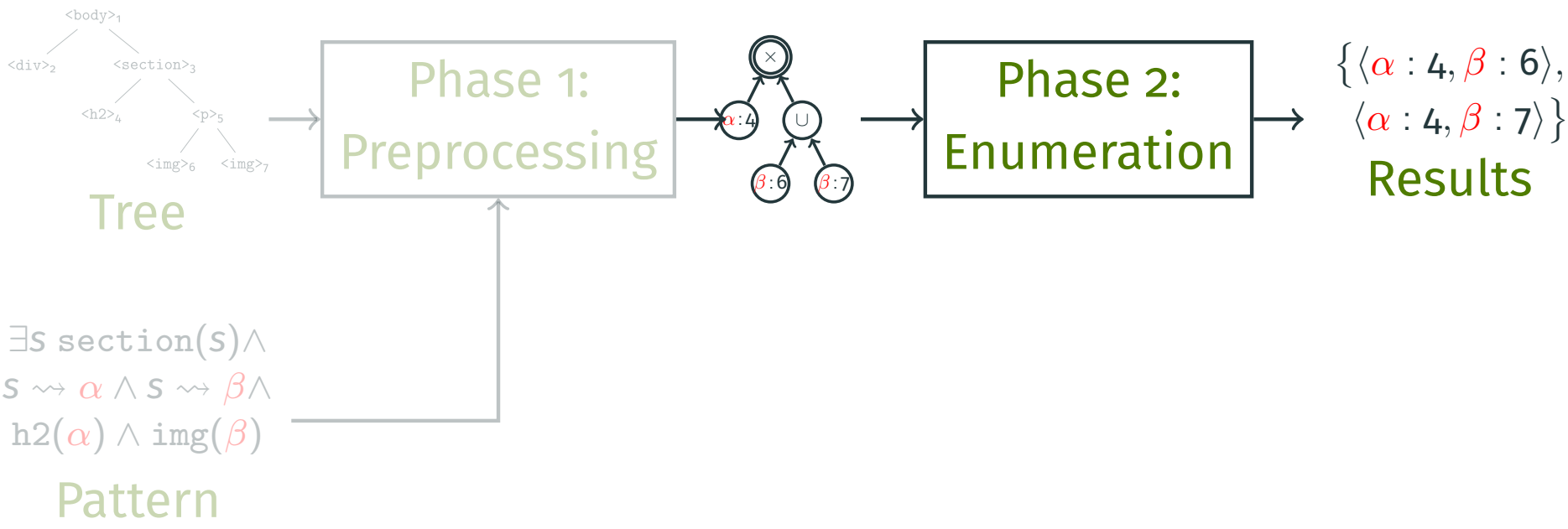
Proof Idea for Trees: Results



Theorem

For any tree automaton A *with capture variables* $\alpha_1, \ldots, \alpha_k$, *given a tree* T, *we can build in* $O(|T| \times |A|)$ *a set circuit capturing exactly the set of tuples* $\{\langle \alpha_1 : n_1, \ldots, \alpha_k : n_k \rangle$ *in the output of* A *on* T

Proof Idea for Trees: Results



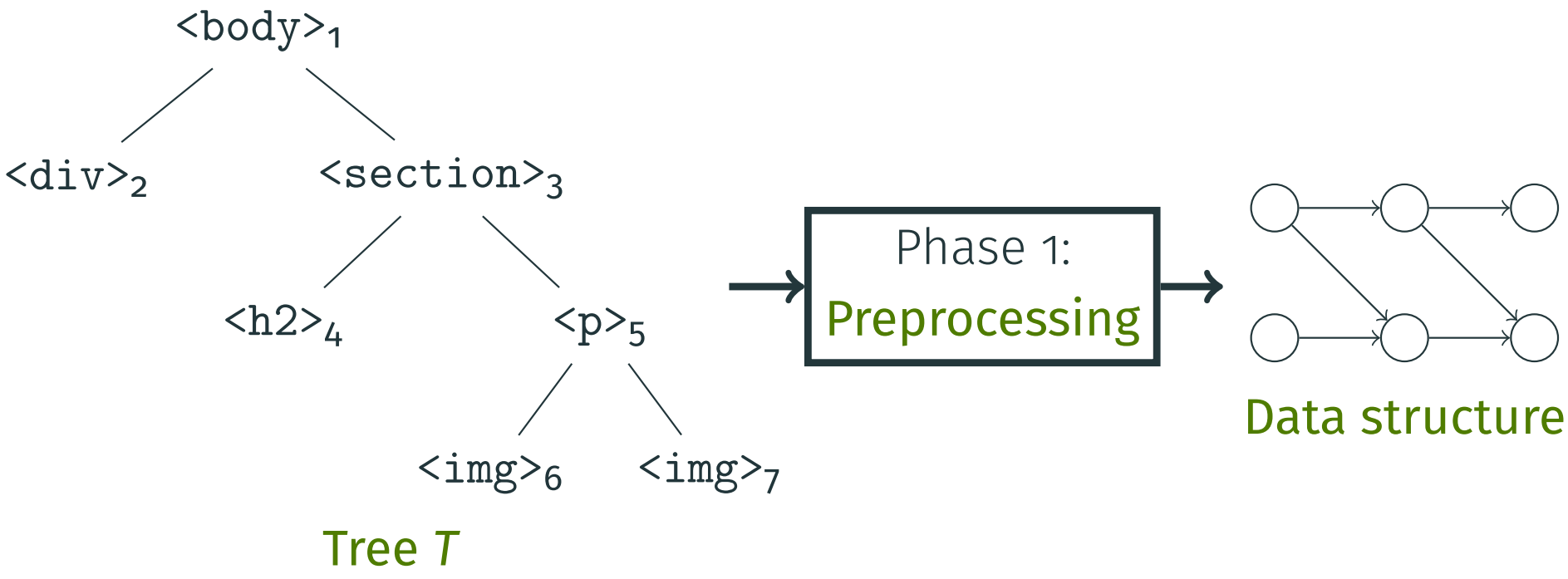
Theorem

Given a set circuit satisfying some conditions, we can enumerate all tuples that it captures with linear preprocessing and constant delay

E.g., for $\{\langle \alpha:4, \beta:6\rangle, \langle \alpha:4, \beta:7\rangle\}$: enumerate $\langle \alpha:4, \beta:6\rangle$ then $\langle \alpha:4, \beta:7\rangle$

Extension: Supporting Updates

Updates



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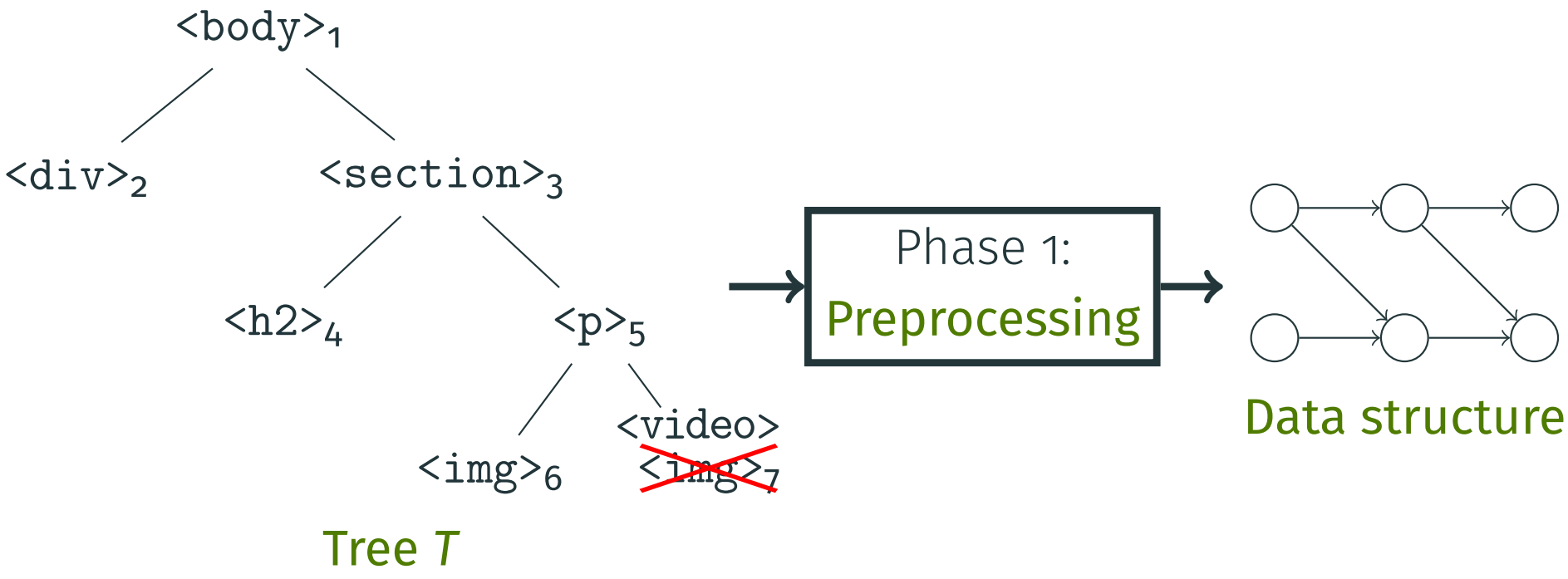

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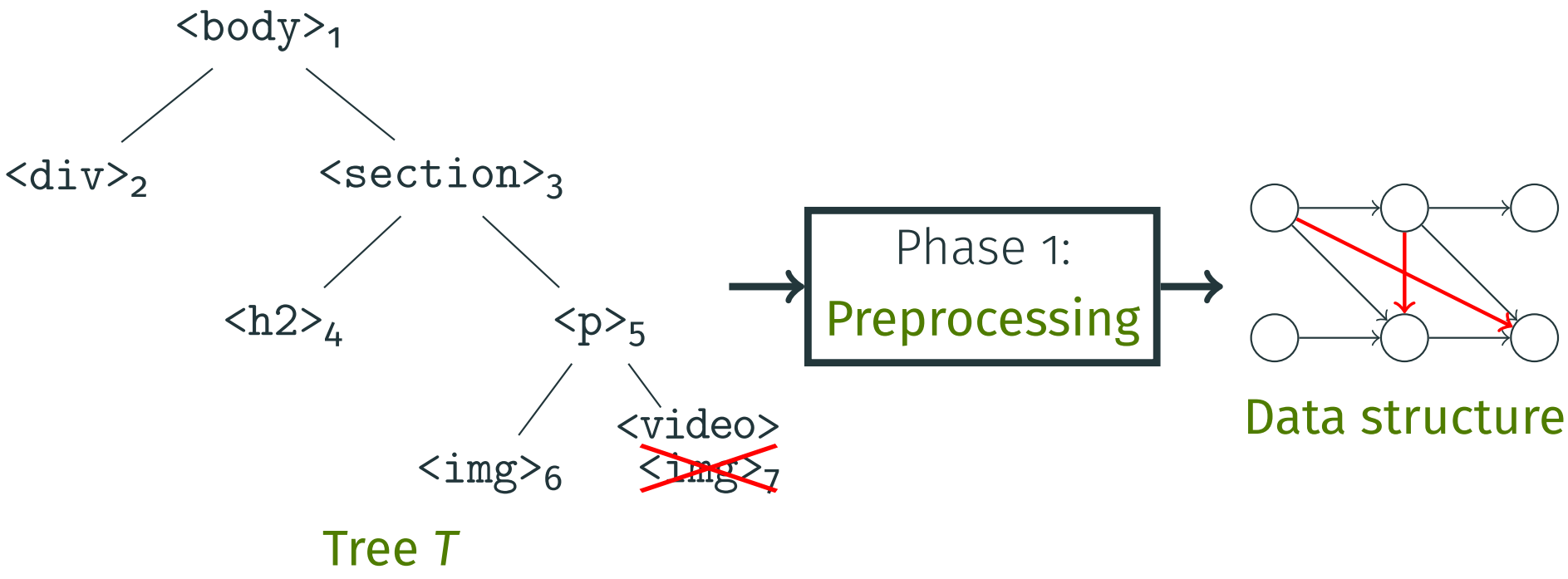

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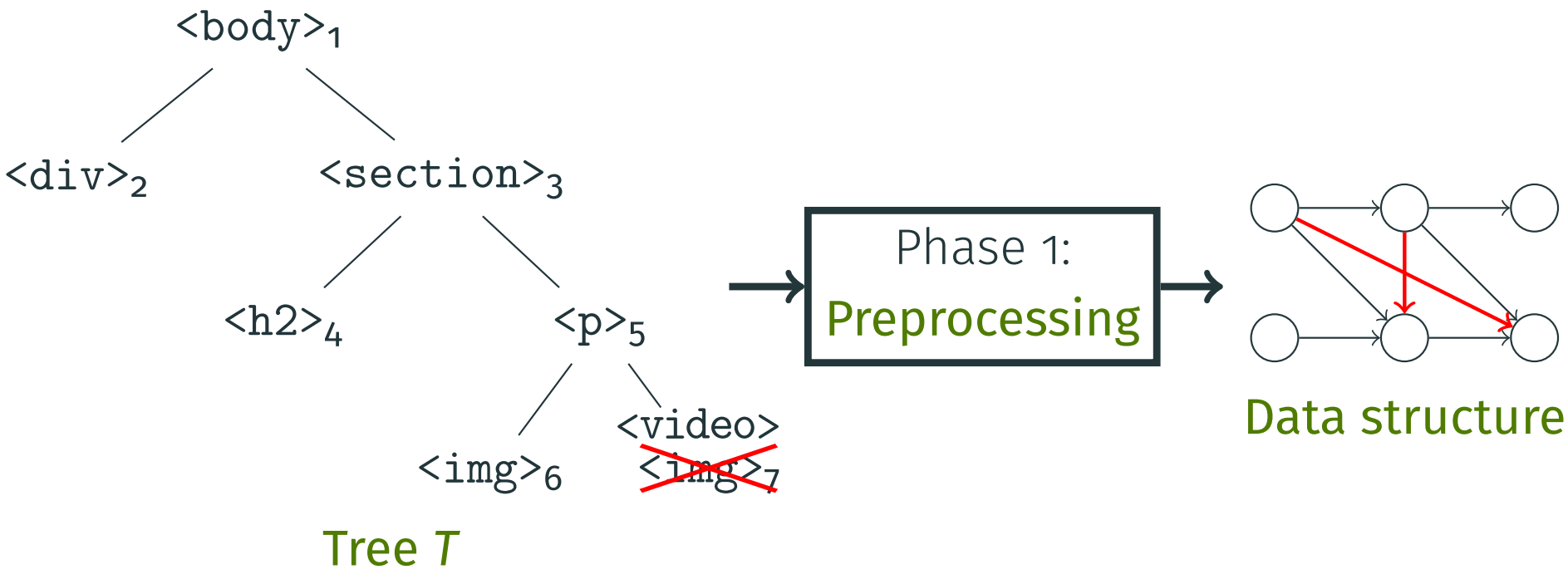

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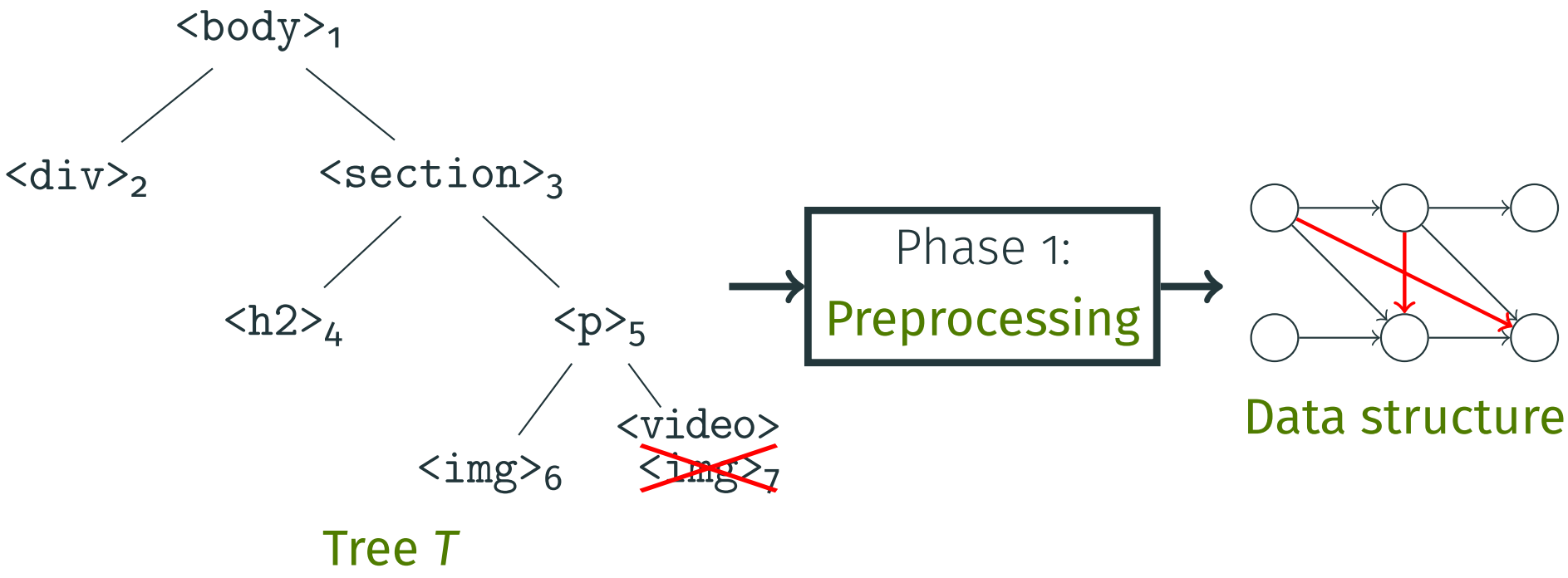

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→ Can we **do better**?

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All these results are on data complexity in T (for a fixed pattern):

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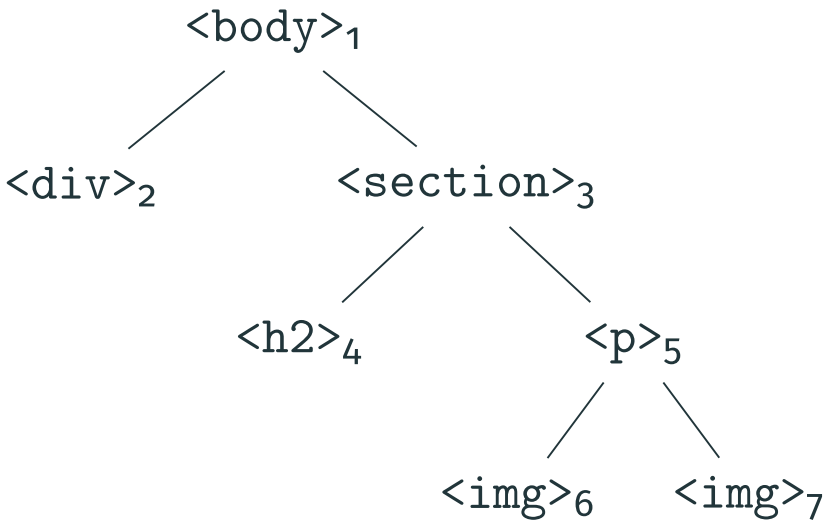

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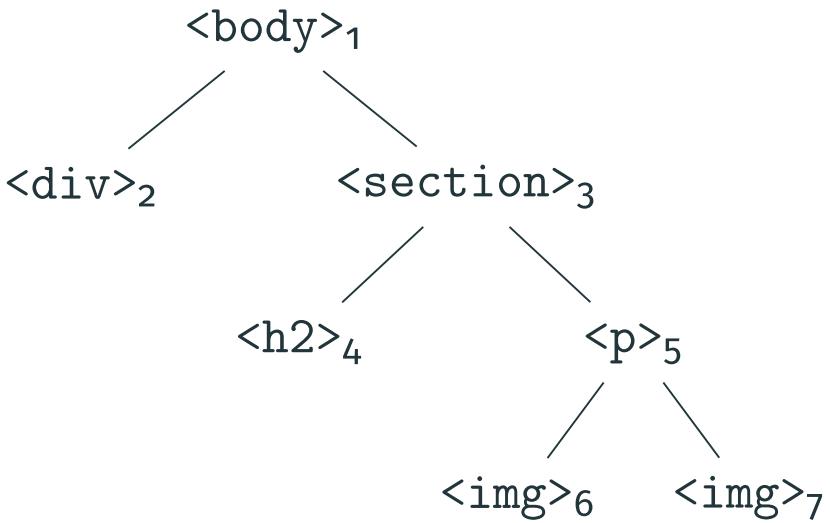

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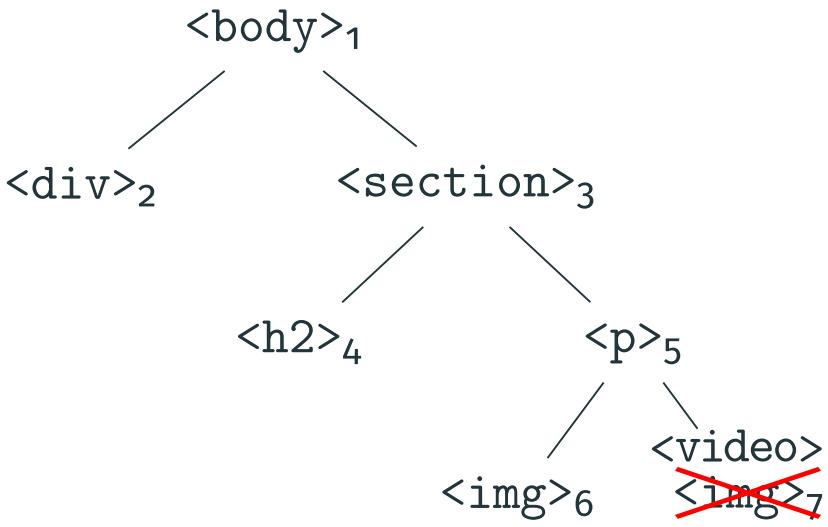

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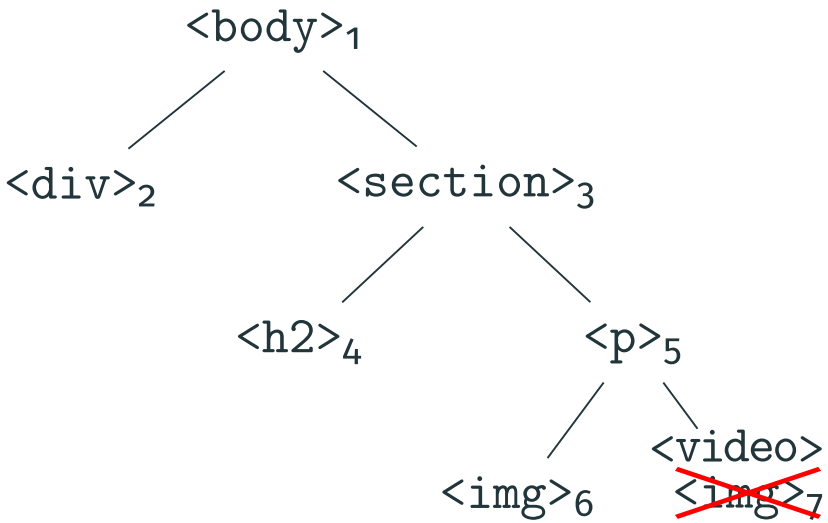

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- Remaining open questions:
  - → Does this hold for more general updates (insert/delete, etc.)?
  - → Can we also achieve tractable combined complexity?

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Thanks for your attention!

References i


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To appear.