Dealing with Similarity in Argumentation

and

Temporal Parametric Semantics in Temporal Markov Logic Network

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Seminar

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Argumentation: Stakes

- A reasoning model based on the need to justify.
  Indispensable for deciding, convincing, explaining, ...
- A multidisciplinary theme
  (**Artificial Intelligence**, Psychology, Linguistics, Philosophy)
- Examples of applications
  - **Medical field**: argumentative diagnostic support system
  - **Legal field**: argued decisions based on law
  - **Online debate systems** (e.g. DebateGraph, Debatepedia)
  - **Online conflict resolution systems** (e.g. CyberSettle)

Argumentation: Process

Given a problem (making a decision, classifying an object, ...)

Input data (e.g. online debates, knowledge base, ...)

↓

Argumentation framework/graph
(Identification of arguments, relations, weights, ...)

↓

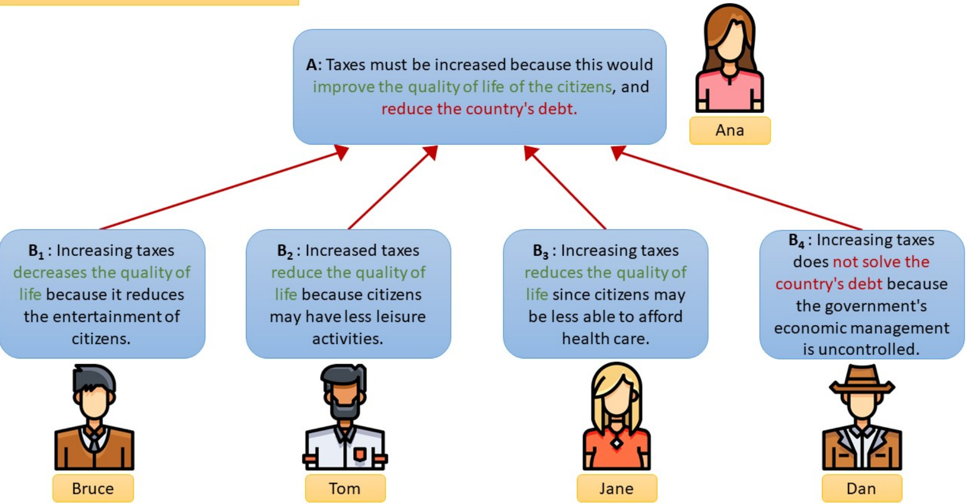
Evaluation of arguments (Semantics)

↓

Output (e.g. winners of a debate,
set of formulas derived from a knowledge base, ...)

Online debate (from a platform)

Should taxes be increased?



Online debate (from a platform)

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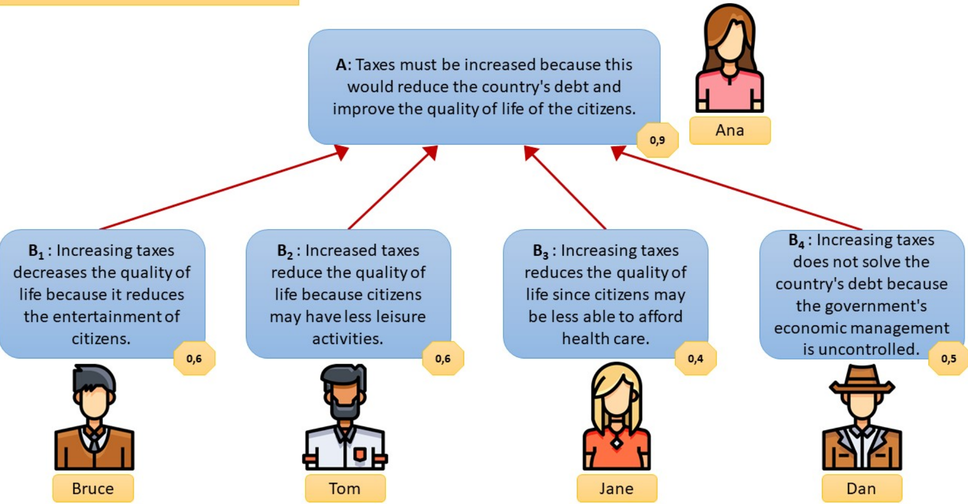

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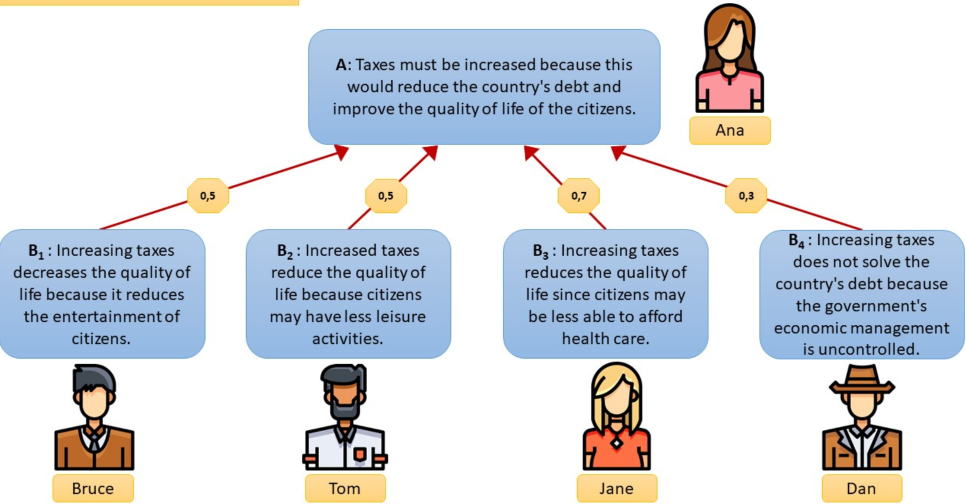

Online debate (from a platform)

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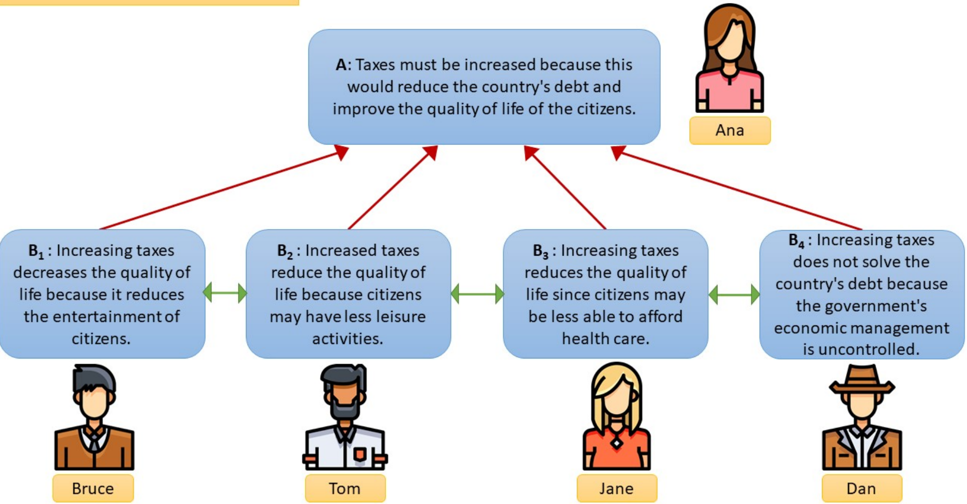


Inconsistency handling by argumentation

$(\mathcal{L}, \vdash)$ is propositional logic
$\Sigma \subseteq_f \mathcal{L}$, i.e. Σ is a finite subset of $\mathcal{L}$
For $\Phi \subseteq \mathcal{L}$; $\mathtt{CN}(\Phi) = \{\psi \in \mathcal{L} \text{ s.t. } \phi \vdash \psi\}$

Definition (Besnard and Hunter (2001))

An argument is a pair $\langle \Phi, \phi \rangle$, where $\Phi \subseteq \Sigma$ and $\phi \in \mathcal{L}$, such that:

- $\Phi \nvdash \bot$ (Consistency)
- $\phi \in \mathtt{CN}(\Phi)$ (Validity)
- $\nexists \Phi' \subset \Phi$ such that $\Phi' \vdash \phi$ (Minimality)

Example ($\Sigma = \{p, q, \neg p, \neg p \rightarrow r\}$)

A:$\langle \{p\}, p \rangle$ **B:**$\langle \{p, q\}, p \wedge q \rangle$ **C:**$\langle \{\neg p, \neg p \rightarrow r\}, r \rangle$ ...

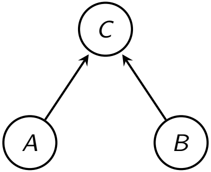
Inconsistency handling by argumentation

Definition (Besnard and Hunter (2001))

An argument $\langle \Phi, \phi \rangle$ defeats an argument $\langle \Psi, \psi \rangle$ iff $\phi \vdash \neg(\psi_1 \wedge \cdots \wedge \psi_n)$ for some $\{\psi_1, \cdots, \psi_n\} \subseteq \Psi$

Example

A:$\langle \{p\}, p \rangle$ **B:**$\langle \{p, q\}, p \wedge q \rangle$ **C:**$\langle \{\neg p, \neg p \rightarrow r\}, r \rangle$



Abstract Argumentation Framework

Definition

An argumentation framework (AF) is a tuple $\langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma \rangle$ s.t.

- $\mathcal{A} \subseteq_f \texttt{Arg}$ [i] (Arguments)
- $\mathbf{w} : \mathcal{A} \rightarrow [0, 1]$ (Weights of arguments)
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ (Attack relation)
- $\sigma : \mathcal{R} \rightarrow [0, 1]$ (Weights of attack relations)

[i] Arg is the universe of all possible arguments

Notation: Let $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma \rangle$ be an AF,
If $\sigma \equiv 1$, then AF is called semi-weighted

Semantics

A semantics is a function $\mathbf{S}$ that assigns to every AF $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma \rangle$,

- a set $\mathtt{Ext}^{\mathbf{S}}_{\mathbf{G}} \in 2^{2^{\mathcal{A}}}$ (Extension-based Semantics)
  look for sets of acceptable arguments, called extensions
- **a weighting $\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}$: $\mathcal{A} \to [0,1]$ (Gradual Semantics)**
  focus on individual arguments
- a preorder $\succeq^{\mathbf{S}}_{\mathbf{G}} \subseteq \mathcal{A} \times \mathcal{A}$ (Ranking-based semantics)
  rank-order arguments from the strongest to the weakest

Semantics

Notation: Let $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma \rangle$ be an $\texttt{AF}$, for $A \in \mathcal{A}$:
$\texttt{Att}(A) = \{B \in \mathcal{A} \mid (B, A) \in \mathcal{R}\}$

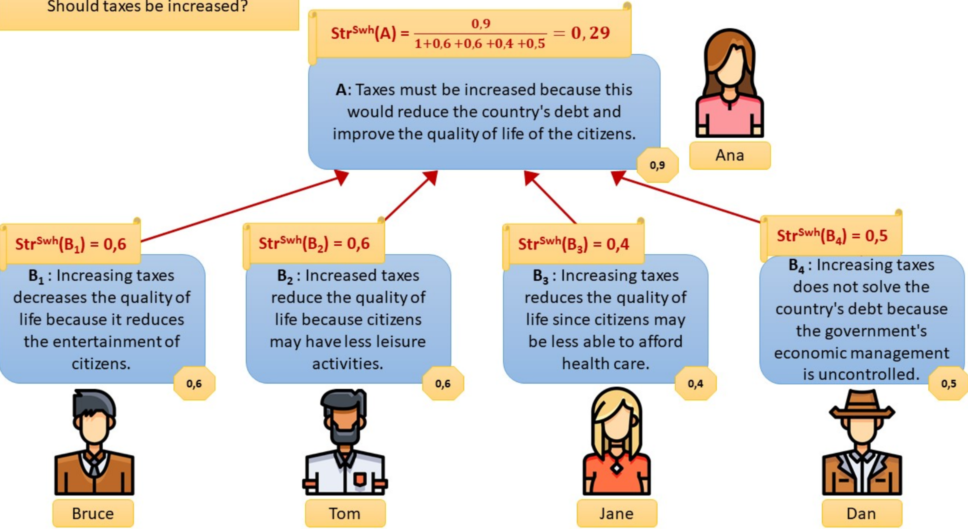
Definition (Amgoud & Doder 2019)

semi-Weighted h-Categoriser is a function $\mathbf{S}_{\texttt{wh}}$ transforming any $\texttt{AF}$ $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma \equiv 1 \rangle$ into a weighting $\texttt{Str}_{\mathbf{G}}^{\mathbf{S}_{\texttt{wh}}}$ on $\mathcal{A}$ such that $\forall A \in \mathcal{A}$,

$$\texttt{Str}_{\mathbf{G}}^{\mathbf{S}_{\texttt{wh}}}(A) = \begin{cases} \mathbf{w}(A) & \text{iff } \texttt{Att}(A) = \emptyset \\ \dfrac{\mathbf{w}(A)}{1 + \sum\limits_{B \in \texttt{Att(A)}} \texttt{Str}_{\mathbf{G}}^{\mathbf{S}_{\texttt{wh}}}(B)} & \text{else} \end{cases}$$

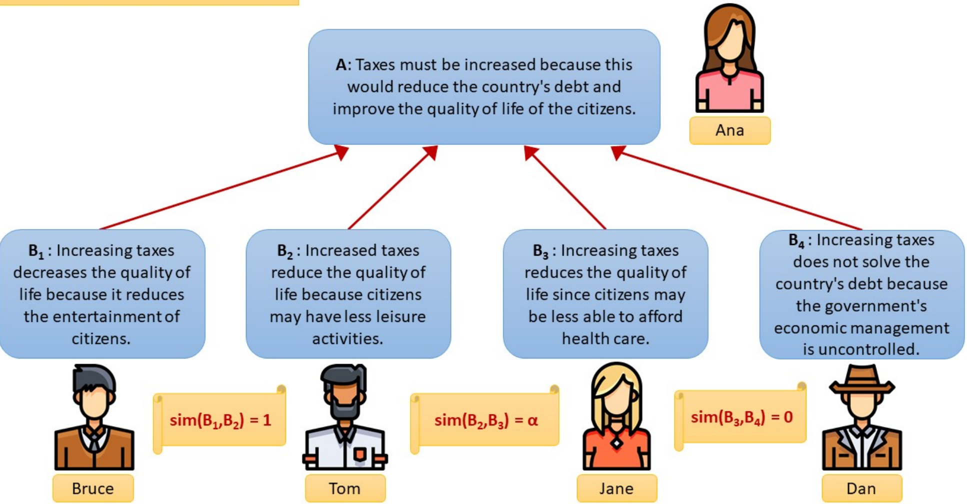
Semantics

Should taxes be increased?



Limits of existing frameworks

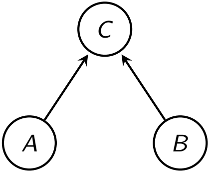
Should taxes be increased?



Limits of existing frameworks

$A : \langle \{p\}, p \rangle \qquad B : \langle \{p, q\}, p \wedge q \rangle \qquad C : \langle \{\neg p, \neg p \rightarrow r\}, r \rangle$

$\mathtt{sim}(A, B) = \alpha$



The **similarity** between the **attackers** should be considered in the **evaluation** of the **attacked** argument

Our Contributions

Two research questions:

1. How to measure similarity between two arguments ?
2. How to define semantics that are able to deal with similarity ?

Outline

1) Introduction
2) Similarity Measures for Logical Arguments
3) Gradual Semantics dealing with Similarity

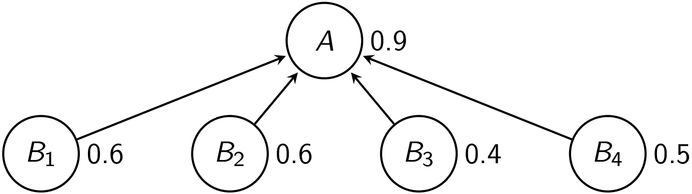
Similarity Measures for Logical Arguments

- Axiomatic Foundations of Similarity Measures [KR18]+ [ECSQARU21]
- Similarity Measures
  - 7 Syntactic Similarity Measures [KR18]
  - 1 Mixed Similarity Measure [ECSQARU19] + [ECSQARU21]
  - 2 Similarity Measures for Non-Concise Arguments [ECSQARU19] + [ECSQARU21]

Gradual Semantics dealing with Similarity

- Evaluation Methods for Gradual Semantics [COMMA20] + [AAAI21]
- Principles for Gradual Semantics [AAAI21]
- Novel Family of Semantics [AAAI21]
- Adjustment Functions [COMMA20] + [AAAI21]

1) Evaluation Methods



1. Assess the strength of the group of attacks on A, $\alpha = \mathbf{g}(0.6, 0.6, 0.4, 0.5)$
2. Evaluate the impact of attacks on the initial weight of A, $\beta = \mathbf{f}(0.9, \alpha)$

1) Evaluation Methods

Definition (Amgoud & Doder, 2018)

An *evaluation method* (EM) is a tuple $\mathbf{M} = \langle \mathbf{f}, \mathbf{g} \rangle$ such that:

- $\mathbf{g} : \bigcup_{k=0}^{+\infty} [0,1]^k \to [0, +\infty[$, such that $\mathbf{g}$ is symmetric
- $\mathbf{f} : [0,1] \times \texttt{Range}(\mathbf{g})$ [a] $\to [0,1]$

[a] $\texttt{Range}(\mathbf{g})$ denotes the co-domain of $\mathbf{g}$

1) Evaluation Methods

Definition (Amgoud & Doder, 2018)

A gradual semantics $\mathbf{S}$ based on an evaluation method $\mathbf{M} = \langle \mathbf{f}, \mathbf{g} \rangle$ assign to every AF $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma \equiv 1 \rangle$ a weighting $\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}$ s.t.
$\forall A \in \mathcal{A}$,
$\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(A) =$

$$\mathbf{f}\Big(\mathbf{w}(A), \mathbf{g}(\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_1), \cdots, \mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_k))\Big)$$

where $\{B_1, \cdots, B_k\} = \mathtt{Att}(A)$

1) Evaluation Methods

Definition (semi-Weighted h-Categoriser)

For any argument A,

$$\mathtt{Str}_{\mathbf{G}}^{\mathbf{S}_{\mathtt{wh}}}(A) = \begin{cases} \mathbf{w}(A) & \text{iff } \mathtt{Att}(A) = \emptyset \\ \dfrac{\mathbf{w}(A)}{1 + \sum\limits_{B \in \mathtt{Att(A)}} \mathtt{Str}_{\mathbf{G}}^{\mathbf{S}_{\mathtt{wh}}}(B)} & \text{else} \end{cases}$$

Example (semi-Weighted h-Categoriser)

$\mathbf{S}_{\mathtt{wh}}$ is based on $\mathbf{M} = \langle \mathbf{f}_{\mathtt{frac}}, \mathbf{g}_{\mathtt{sum}} \rangle$ such that:

$$\begin{cases} \mathbf{f}_{\mathtt{frac}}(x_1, x_2) = \frac{x_1}{1+x_2} \\ \mathbf{g}_{\mathtt{sum}}(x_1, \cdots, x_n) = \sum\limits_{i=1}^{n} x_i \end{cases}$$

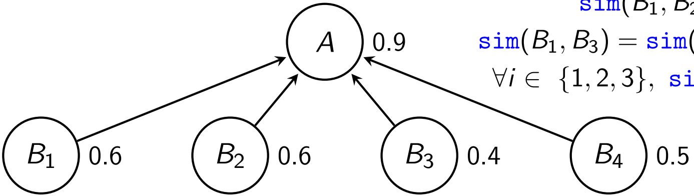
1.a) New Argumentation Framework [AAAI21]

Definition (SSWAF)

A semi-weighted argumentation framework extended by a similarity measure (SSWAF) is a tuple $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma \equiv 1, \mathtt{sim} \rangle$, where

- $\mathcal{A} \subseteq_f \mathtt{Arg}$
- $\mathbf{w} : \mathcal{A} \rightarrow [0, 1]$
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$
- $\sigma : \mathcal{R} \rightarrow 1$
- $\mathtt{sim} : (\mathcal{A} \times \mathcal{A}) \rightarrow [0, 1]$ (Similarity measure)

1.b) Extended Evaluation Methods [AAAI21]



$\texttt{sim}(B_1, B_2) = 1$

$\texttt{sim}(B_1, B_3) = \texttt{sim}(B_2, B_3) = 0.25$

$\forall i \in \{1, 2, 3\},\ \texttt{sim}(B_i, B_4) = 0$

1. Adjust the strength of every attack w.r.t. similarity,
   $\mathbf{n}((\texttt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_1), B_1), (\texttt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_2), B_2), (\texttt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_3), B_3), (\texttt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_4), B_4)) = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$
2. Assess the strength of the group of attacks on A,
   $\beta = \mathbf{g}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$
3. Evaluate the impact of attacks on the initial weight of A,
   $\delta = \mathbf{f}(0.9, \beta)$

1.b) Extended Evaluation Methods [AAAI21]

Definition (EM)

An *evaluation method* (EM) is a tuple $\mathbf{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{n} \rangle$ such that:

- $\mathbf{f} : [0,1] \times \texttt{Range}(\mathbf{g}) \rightarrow [0,1]$ (influence function)
- $\mathbf{g} : \bigcup_{k=0}^{+\infty} [0,1]^k \rightarrow [0, +\infty[$ (aggregation function)
- $\mathbf{n} : \bigcup_{k=0}^{+\infty} ([0,1] \times \texttt{Arg})^k \rightarrow [0,1]^k$ (adjustment function)

1.b) Extended Evaluation Methods [AAAI21]

Definition (S based on M)

A *gradual semantics* **S** based on an evaluation method $\mathbf{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{n} \rangle$ is a function transforming every SSWAF $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma \equiv 1, \mathtt{sim} \rangle$ into a weighting $\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}} : \mathcal{A} \to [0, 1]$ such that for every $A \in \mathcal{A}$,

$\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(A) =$

$$\mathbf{f}\left(\mathbf{w}(A), \mathbf{g}\left(\mathbf{n}\left((\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_1), B_1), \cdots, (\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_k), B_k)\right)\right)\right),$$

where $\{B_1, \cdots, B_k\} = \mathtt{Att}(A)$

1.c) Well-behaved Adjustment Function $\mathbf{n}$ [AAAI21]

Let $x_i, y_i \in \mathbb{R}$ and $A_i, B_i \in \texttt{Arg}$, $\mathbf{n}$ is *well-behaved* iff:

(a) $\mathbf{n}() = ()$,

(b) $\mathbf{n}((x, A)) = (x)$,

(c) $\mathbf{g}(\mathbf{n}((x_1, A_1), \cdots, (x_k, A_k))) \leq \mathbf{g}(\mathbf{n}((x_1, B_1), \cdots, (x_k, B_k)))$ if $\forall i, j \in \{1, \cdots, k\}$ $i \neq j$, $\texttt{sim}(A_i, A_j) \geq \texttt{sim}(B_i, B_j)$,

(d) If $\exists i \in \{1, \cdots, k\}$ s.t. $x_i > 0$ then $\mathbf{g}(\mathbf{n}((x_1, A_1), \cdots, (x_k, A_k))) > 0$,

(e) $\mathbf{g}(\mathbf{n}((x_1, A_1), \cdots, (x_k, A_k))) \leq \mathbf{g}(\mathbf{n}((y_1, A_1) \cdots, (y_k, A_k)))$ if $\forall i \in \{1, \cdots, k\}$, $x_i \leq y_i$,

(f) $\mathbf{n}$ is symmetric,

(g) $\mathbf{n}((x_1, A_1), \cdots, (x_{k+1}, A_{k+1})) = (\mathbf{n}((x_1, A_1), \cdots, (x_k, A_k)), x_{k+1})$ if $\forall i \in \{1, \ldots, k\}$, $\texttt{sim}(A_i, A_{k+1}) = 0$.

1.c) Well-behaved Adjustment Function $\mathbf{n}$ [AAAI21]

Only one semantics based on an EM

Theorem

Let $\mathbf{M}^*$ *be the set of all well-behaved evaluation methods* $\mathbf{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{n} \rangle$ *such that:*

- $\lim_{x_2 \to x_0} \mathbf{f}(x_1, x_2) = \mathbf{f}(x_1, x_0), \forall x_0 \neq 0.$
- $\lim_{x \to x_0} \mathbf{g}(x_1, \cdots, x_k, x) = \mathbf{g}(x_1, \cdots, x_k, x_0), \forall x_0 \neq 0.$
- $\mathbf{n}$ *is continuous on each numerical variable.*
- $\lambda \mathbf{f}(x_1, \lambda x_2) < \mathbf{f}(x_1, x_2), \forall \lambda \in [0, 1[, x_1 \neq 0.$
- $\mathbf{g}(\mathbf{n}(\lambda x_1, \cdots, \lambda x_k, B_1, \cdots, B_k)) \geq \lambda \mathbf{g}(\mathbf{n}(x_1, \cdots, x_k, B_1, \cdots, B_k)), \forall \lambda \in [0, 1].$

For any $\mathbf{M} \in \mathbf{M}^*$*, for all gradual semantics* $\mathbf{S}, \mathbf{S}'$*,*

if $\mathbf{S}, \mathbf{S}'$ *are based on* $\mathbf{M}$*, then* $\mathbf{S} \equiv \mathbf{S}'$

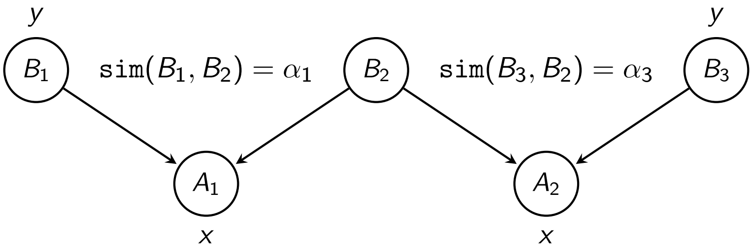
2) Principles

[AAAI21]

Principles for gradual semantics dealing with similarity:

- Neutrality
- (Strict) Monotony
- (Strict) Reinforcement
- (Strict) Sensitivity to Similarity
- ...

2) Principles [AAAI21]



(Strict) Sensitivity to Similarity: the greater the similarities between attackers of an argument, the stronger the argument; if $\alpha_1 \geq \alpha_3$ then $\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(A_1) \geq \mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(A_2)$

2) Principles

[AAAI21]

Theorem

Let **S** *be a gradual semantics based on an* EM **M**. *If* **M** *is well-behaved, then* **S** *satisfies Reinforcement, Monotony, Neutrality and Sensitivity to Similarity*

3 additional constraints ensure the satisfaction of the strict versions.

For instance, Strict Monotony is satisfied when:

$$\mathbf{g}(x_1, \cdots, x_k, y) < \mathbf{g}(x_1, \cdots, x_k, z) \qquad \text{if } y < z \qquad \textbf{(C3)}$$

3) Novel Family of Semantics [AAAI21]

Definition ($\mathbf{S}^*$)

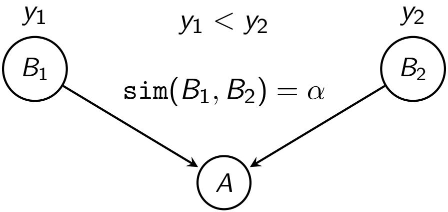
We define by $\mathbf{S}^*$ the set of all semantics that are based on an evaluation method from $\mathbf{M}^*$

Theorem

Any gradual semantics $\mathbf{S} \in \mathbf{S}^*$ satisfies Reinforcement, Monotony, Neutrality and Sensitivity to Similarity

4) Adjustment Functions

[COMMA20] + [AAAI21]



- *Conjunctive*: **n** removes the redundancy from the weakest argument (B_1)
- *Disjunctive*: **n** removes the redundancy from the strongest argument (B_2)
- *Compensative*: **n** distributes the burden to both

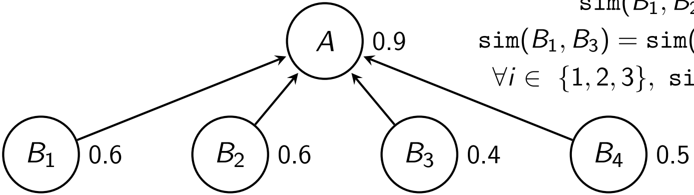
4.a) Adjustment Function $\mathbf{n}^{\rho}_{\mathtt{max}}$ [AAAI21]

Definition (Parameterised Function $\mathbf{n}^{\rho}_{\mathtt{max}}$)

Let $A_1, \cdots, A_k \in \mathtt{Arg}$, $x_1, \cdots, x_k \in [0,1]$, and ρ a fixed permutation on the set $\{1, \cdots, k\}$ such that if $x_{\rho(i)} = 0$ then $x_{\rho(i+1)} = 0$ $\forall i < k$, or $i = k$. $\mathbf{n}^{\rho}_{\mathtt{max}}() = ()$, otherwise:

$$\mathbf{n}^{\rho}_{\mathtt{max}}((x_1, A_1), \cdots, (x_k, A_k)) = \Big(x_{\rho(1)},\; x_{\rho(2)} \times (1 - \mathtt{max}(\mathtt{sim}(A_{\rho(1)}, A_{\rho(2)}))),\; \cdots,\; x_{\rho(k)} \times (1 - \mathtt{max}(\mathtt{sim}(A_{\rho(1)}, A_{\rho(k)}), \cdots, \mathtt{sim}(A_{\rho(k-1)}, A_{\rho(k)}))) \Big)$$

4.a) Adjustment Function $\mathbf{n}^{\rho}_{\mathtt{max}}$ [AAAI21]



$$\mathtt{sim}(B_1, B_2) = 1$$
$$\mathtt{sim}(B_1, B_3) = \mathtt{sim}(B_2, B_3) = 0.25$$
$$\forall i \in \{1, 2, 3\},\ \mathtt{sim}(B_i, B_4) = 0$$

Let ρ_{dec} rank arguments in a decreasing and unique order based on their strength, and $\mathbf{S}$ s.t. $\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_i) = \mathbf{w}(B_i)$ for $i \in \{1, 2, 3, 4\}$

Then $\mathbf{n}^{\rho_{dec}}_{\mathtt{max}}$ is a conjunctive adjustment function and

$$\mathbf{n}^{\rho_{dec}}_{\mathtt{max}}((\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_1), B_1), (\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_2), B_2), (\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_3), B_3), (\mathtt{Str}^{\mathbf{S}}_{\mathbf{G}}(B_4), B_4)) = (0.6, 0.6 \times (1-1), 0.5 \times (1-0), 0.4 \times (1-0.25)) = (0.6, 0, 0.5, 0.3)$$

4.b) Results

[AAAI21] + [New]

Proposition

Let $\mathbf{f}, \mathbf{g}$ *be well-behaved functions and* $\mathbf{g}$ *satisfies the following property:*

let $\lambda \in [0,1]$, $x_1, \cdots, x_k \in [0,1]$, *then* $\mathbf{g}(\lambda x_1, \cdots, \lambda x_k) \geq \lambda \mathbf{g}(x_1, \cdots, x_k)$.

The following properties hold:

- $\mathbf{n}^{\rho}_{\mathtt{max}}$ *and* $\mathbf{n}_{\mathtt{wh}}$ *are well-behaved*
- *for all functions* $\mathbf{f}, \mathbf{g}$ *that are well-behaved, it holds that* $\langle \mathbf{f}, \mathbf{g}, \mathbf{n}^{\rho}_{\mathtt{max}} \rangle, \langle \mathbf{f}, \mathbf{g}, \mathbf{n}_{\mathtt{wh}} \rangle \in \mathbf{M}^*$

5) Illustration of Gradual Semantics [COMMA20] + [AAAI21]

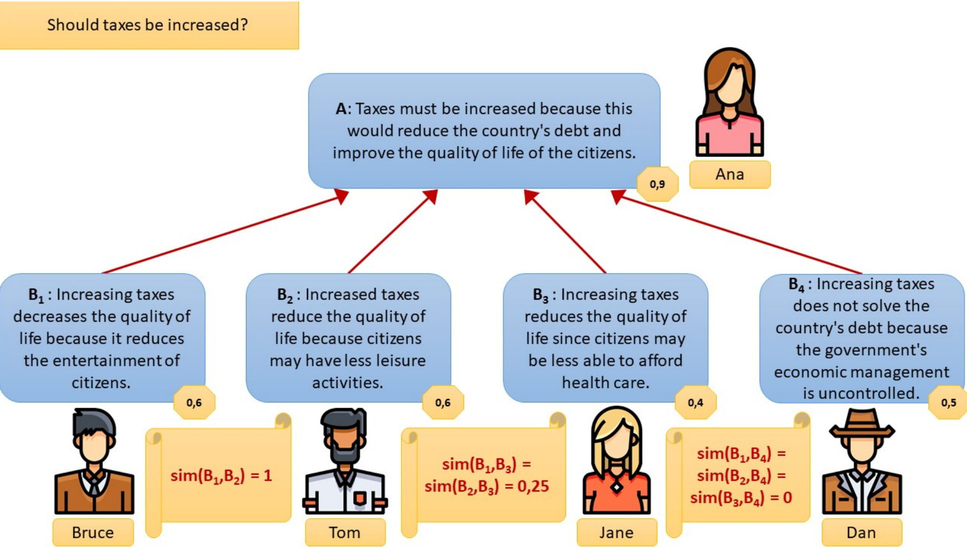
Definition ($\mathbf{S^n}$)

Semantics $\mathbf{S^n}$ based on $\langle \mathbf{f}_{\mathtt{frac}}, \mathbf{g}_{\mathtt{sum}}, \mathbf{n} \rangle$ is a function transforming any SSWAF $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma \equiv 1, \mathtt{sim} \rangle$ into a function $\mathtt{Str}^{\mathbf{n}}$ from $\mathcal{A}$ to $[0,1]$ s.t. $\forall A \in \mathcal{A}$, $\mathtt{Str}^{\mathbf{n}}(A) =$

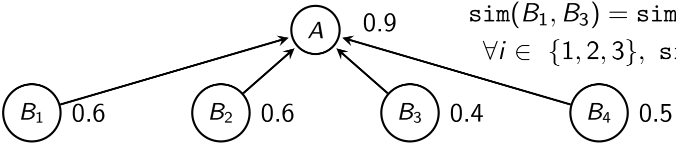
$$\frac{\mathbf{w}(A)}{1 + \sum_{i=1}^{k} \left(\mathbf{n}\Big((\mathtt{Str}^{\mathbf{n}}(B_1), B_1), \cdots, (\mathtt{Str}^{\mathbf{n}}(B_k), B_k) \Big) \right)}$$

where $\mathtt{Att}(A) = \{B_1, \cdots, B_k\}$. If $\mathtt{Att}(A) = \emptyset$, then $\sum_{i=1}^{k}(.) = 0$.

5) Illustration of Gradual Semantics [COMMA20] + [AAAI21]



5) Illustration of Gradual Semantics [COMMA20] + [AAAI21]



$$\texttt{sim}(B_1, B_2) = 1$$
$$\texttt{sim}(B_1, B_3) = \texttt{sim}(B_2, B_3) = 0.25$$
$$\forall i \in \{1, 2, 3\},\ \texttt{sim}(B_i, B_4) = 0$$

Without similarity: $\texttt{Str}^{\mathbf{S}_{\texttt{wh}}}(A) = 0.29$

With similarity:

$$\texttt{Str}^{\mathbf{n}_{\texttt{max}}^{\rho_{\texttt{dec}}}}(A) = \frac{0.9}{1 + 0.6 + 0 + 0.5 + 0.3} = \frac{0.9}{2.4} = 0.375$$

$$\texttt{Str}^{\mathbf{n}_{\texttt{wh}}}(A) = \frac{0.9}{1 + 0.404 + 0.404 + 0.5 + 0.333} = \frac{0.9}{2.641} = 0.341$$

5) Illustration of Gradual Semantics [COMMA20] + [AAAI21]

Theorem

For any ρ, it holds that $\mathbf{S}^{\mathbf{n}^{\rho}_{\mathtt{max}}} \in \mathbf{S}^*$ and $\mathbf{S}^{\mathbf{n}^{\rho}_{\mathtt{max}}}$ satisfies:

- *Neutrality*
- *(Strict) Monotony*
- *Reinforcement*
- *Sensitivity to Similarity*

The semantics $\mathbf{S}^{\mathbf{n}_{\mathtt{wh}}}$ satisfies all the principles and $\mathbf{S}^{\mathbf{n}_{\mathtt{wh}}} \in \mathbf{S}^*$

Conclusion

Two research questions with almost no work in the literature:

1. **How to measure similarity between two arguments?**
   - Proposition of principles for similarity measures
   - Proposition of various similarity measures

2. **How to define semantics that deal with similarity?**
   - Extension of evaluation methods with a novel adjustment function
   - Proposition of principles for evaluation methods and semantics dealing with similarity
   - Proposition of a broad family of gradual semantics encompassing almost all the existing gradual semantics
   - Proposition of different adjustment functions

References & Questions

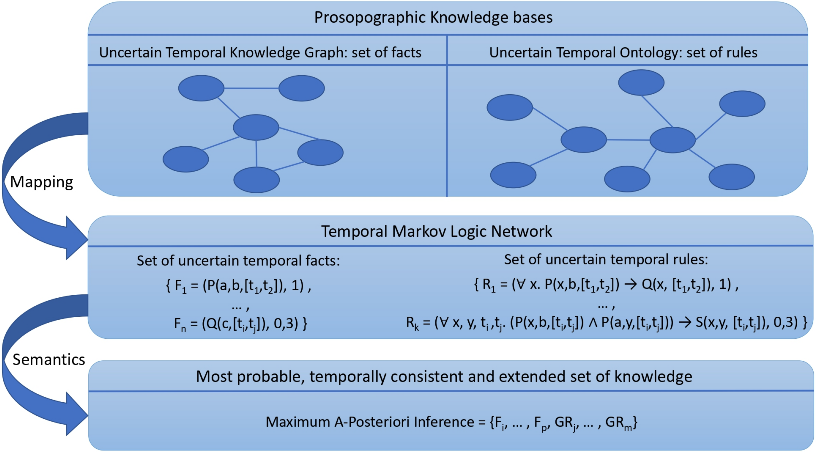
Thank you for your attention

References:

- ✓ KR-2018: L. Amgoud, V. David, Measuring Similarity between Logical Arguments
- ✓ ECSQARU-2019: L. Amgoud, V. David, D. Doder, Similarity Measures between Arguments Revisited
- ✓ COMMA-2020: L. Amgoud, V. David, An Adjustment Function for Dealing with Similarities
- ✓ AAAI-2021: L. Amgoud, V. David, A General Setting for Gradual Semantics Dealing with Similarities
- ✓ ECSQARU-2021: L. Amgoud, V. David, Similarity measures based on compiled arguments

Questions ?

Context of the Daphne Project



MAP Inferences are useful for answering historical queries, such as validating historical assumptions

First Order Logic

Definition (FOL)

FOL is a set of formulae built up from :

- constants ($\{a, b, c, \cdots\} \in \mathbf{C}$),
- variables ($\{x, y, z, \cdots\} \in \mathbf{V}$),
- functions ($\{f, g, h, \cdots\} \in \mathbf{F}$),
- predicates ($\{P, Q, R, \cdots\} \in \mathbf{P}$),
- connectives ($\neg$, $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$),
- quantifier symbols ($\forall, \exists$).

Where $\{\phi, \psi, \cdots\} \in \texttt{FOL}$ are formula and $\{\Phi, \Psi, \cdots\} \subseteq \texttt{FOL}$ are subset of formulae.

A grounded formula is a formula without any variable.

Temporal Predicate and Formula

Definition (TP an TF)

Let a set of first order formulae FOL and a time interval $\mathcal{T} \subset \mathbf{C}$, a temporal predicate $TP \in \mathbf{P}$ is an extension of a simple predicate $P \in \mathbf{P}$ iff $P(x_1, \cdots, x_n)$ and $TP(x_1, \cdots, x_n, T)$, where $T = [t, t'] \subseteq \mathcal{T}$.

Notation: TP (resp. TF) is the set of *temporal predicates* (resp. the set of *temporal formulae*).

Temporal Markov Logic Network

Definition (TMLN)

A Temporal Markov Logic Network $\mathbf{M} = (\mathbf{F}, \mathbf{R})$ is a set of weighted temporal facts and rules where $\mathbf{F}$ and $\mathbf{R}$ are sets of pairs such that:

- $\mathbf{F} = \{(\phi_1, w_1), \cdots, (\phi_n, w_n)\}$ with $\forall i \in \{1, \cdots, n\}$, $\phi_i \in \mathtt{TF}$ such that it is grounded and $w_i \in [0, 1]$,
- $\mathbf{R} = \{(\phi'_1, w'_1), \cdots, (\phi'_k, w'_k)\}$ with $\forall i \in \{1, \cdots, k\}$, $\phi'_i \in \mathtt{TF}$ such that it is not grounded and in the form (premises,conclusion), i.e. $(\psi_1 \wedge \cdots \wedge \psi_l) \rightarrow \psi_{l+1}$ where $\forall j \in \{1, \cdots, l+1\}$, $\psi_j \in \mathtt{TF}$, and $w_i \in [0, 1]$.

The universe of all TMLNs is denoted by $\mathtt{TMLN}$.

Temporal Markov Logic Network

Example of TMLN for *Nicole Oresme*:

F_1 : $(Person(NO, [1320, 1382])$ $, 1)$
F_2 : $(Philosopher(NO, [1320, 1382])$ $, 1)$
F_3 : $(LivePeriod(NO, MiddleAges, [1320, 1382])$ $, 1)$
F_4 : $(Studied(NO, CollegeOfNavarre, [1340, 1356])$ $, 0.7)$
F_5 : $(\neg Studied(NO, CollegeOfNavarre, [1350, 1360])$ $, 0.3)$
F_6 : $(\neg Studied(NO, CollegeOfNavarre, [1350, 1355])$ $, 0.4)$
R_1 : $(\forall x, T, (Person(x, T) \wedge LivePeriod(x, MiddleAges, T) \wedge$
$Studied(x, CollegeOfNavarre, T)) \rightarrow PeasantFamily(x, T)$ $, 0.8)$
R_2 : $\forall x, T, (Philosopher(x, T) \wedge LivePeriod(x, MiddleAges, T))$
$\rightarrow \neg PeasantFamily(x, T)$ $, 0.6)$

Temporal Markov Logic Network

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Temporal Maximum A-Posteriori Inference

Definition

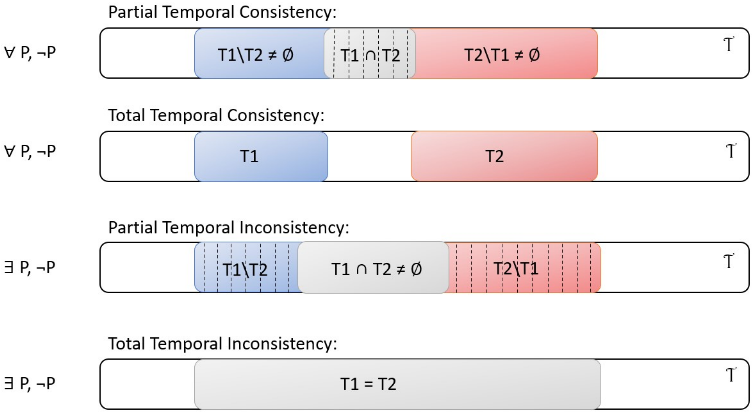
Temporal MAP inference in TMLN corresponds to obtaining the most probable, temporally consistent, and expanded state.

Given $\mathbf{M} \in \texttt{TMLN}$ and $S \in \texttt{Sem}$ computing the score of an interpretation I, a method solving a MAP problem is denoted by "$\texttt{map} : \texttt{TMLN} \times \texttt{Sem} \rightarrow \mathcal{P}^{a}(\texttt{TMLN}^{*})$", *s.t.*:

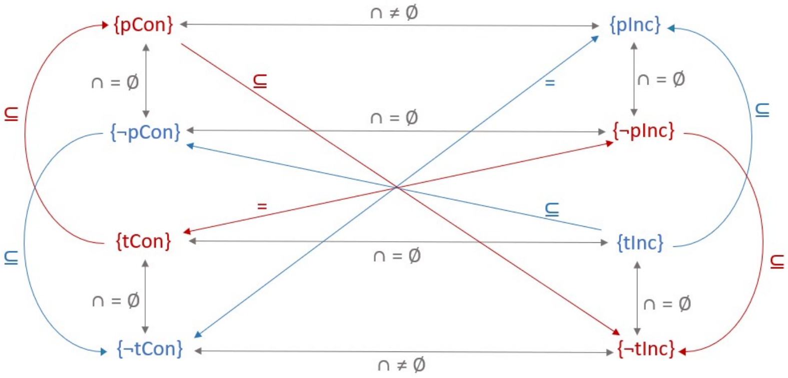
$$\texttt{map}(\mathbf{M}, S) = \{I \mid I \in \underset{I \subseteq \texttt{MB}(\mathbf{M})}{\text{argmax}} \; S(I) \text{ and }$$
$$\nexists I' \in \underset{I' \subseteq \texttt{MB}(\mathbf{M})}{\text{argmax}} \; S(I') \text{ s.t. } I \subset I'\}$$

[a] $\mathcal{P}(X)$ denote the powerset of X

Temporal In/Consistency



Temporal In/Consistency



Temporal Parametric Semantics (TPS)

Definition (Temporal Parametric Semantics)

A *temporal parametric semantics* is a tuple $\texttt{TPS} = \langle \Delta, \sigma, \Theta \rangle$, *s.t.*:

- $\Delta : \texttt{TMLN}^* \to \{0, 1\}$,
- $\sigma : \texttt{TMLN}^* \to \bigcup_{k=0}^{+\infty} [0,1]^k$.
- $\Theta : \bigcup_{k=0}^{+\infty} [0,1]^k \to [0, +\infty[$,

For any $\mathbf{M} \in \texttt{TMLN}$, $I \subseteq \texttt{MB}(\mathbf{M})$, the strength of a temporal parametric semantics $\texttt{TPS} = \langle \Delta, \sigma, \Theta \rangle$ is computed by:

$$\texttt{TPS}(I) = \Delta(I) \cdot \Theta\Big(\sigma(I)\Big).$$

Temporal Parametric Semantics (TPS)

Theorem

Let $\mathbf{M} \in \mathtt{TMLN}$, *for any* σ *and* Θ, *we denote by:*

- $\mathtt{TPS}_{\mathtt{tCon}} = \langle \Delta_{\mathtt{tCon}}, \sigma, \Theta \rangle$, $\mathtt{TPS}_{\mathtt{pInc}} = \langle \Delta_{\mathtt{pInc}}, \sigma, \Theta \rangle$,
- $\mathtt{TPS}_{\mathtt{pCon}} = \langle \Delta_{\mathtt{pCon}}, \sigma, \Theta \rangle$, $\mathtt{TPS}_{\mathtt{tInc}} = \langle \Delta_{\mathtt{tInc}}, \sigma, \Theta \rangle$.

Hence: $\forall I_{tc} \in \mathtt{map}(\mathbf{M}, \mathtt{TPS}_{\mathtt{tCon}})$, $\forall I_{pi} \in \mathtt{map}(\mathbf{M}, \mathtt{TPS}_{\mathtt{pInc}})$, $\forall I_{pc} \in \mathtt{map}(\mathbf{M}, \mathtt{TPS}_{\mathtt{pCon}})$, $\forall I_{ti} \in \mathtt{map}(\mathbf{M}, \mathtt{TPS}_{\mathtt{tInc}})$,

$$\mathtt{TPS}_{\mathtt{tCon}}(I_{tc}) = \mathtt{TPS}_{\mathtt{pInc}}(I_{pi}) \leq \mathtt{TPS}_{\mathtt{pCon}}(I_{pc}) \leq \mathtt{TPS}_{\mathtt{tInc}}(I_{ti}).$$

Temporal Parametric Semantics (TPS)

Definition (Selective Functions)

Let $\mathbf{M} \in \texttt{TMLN}$, $\{(\phi_1, w_1), \cdots, (\phi_n, w_n)\} \subseteq \texttt{MB}(\mathbf{M})$:

- $\sigma_{id}(\{(\phi_1, w_1), \cdots, (\phi_n, w_n)\}) = (w_1, \cdots, w_n)$
- $\sigma_{thresh,\alpha}(\{(\phi_1, w_1), \cdots, (\phi_n, w_n)\}) =$ $(\texttt{max}(w_1 - \alpha, 0), \cdots, \texttt{max}(w_n - \alpha, 0))$ s.t. $\alpha \in [0, 1[$
-
  - let $\phi = (\psi_1 \wedge \cdots \wedge \psi_k) \rightarrow \psi_{k+1}$ a rule, $\texttt{prem}(\phi) = \{\psi_1, \cdots, \psi_k\}$.
  - $\texttt{imp}((\phi, w), \{(\phi_1, w_1), \cdots, (\phi_n, w_n)\}) =$
    $$\begin{cases} 0 & \text{if } \phi \text{ is a grounded rule s.t. } \exists \psi_i \in \texttt{prem}(\phi) \\ & \text{s.t. } \psi_i \notin \texttt{CN}(\{\phi_1, \cdots, \phi_n\}) \\ w & \text{otherwise} \end{cases}$$
  - $\sigma_{rule}(\{(\phi_1, w_1), \cdots, (\phi_n, w_n)\}) =$
    $$\Big(\texttt{imp}((\phi_1, w_1), \{(\phi_2, w_2), \cdots, (\phi_n, w_n)\}), \cdots, \texttt{imp}((\phi_n, w_n), \{(\phi_1, w_1), \cdots, (\phi_{n-1}, w_{n-1})\})\Big)$$

Temporal Parametric Semantics (TPS)

Definition (Aggregate Functions)

Let $\{w_1, \cdots, w_n\}$ such that $n \in [0, +\infty[$ and $\forall i \in [0, n]$, $w_i \in [0, 1]$.

- $\Theta_{sum}(w_1, \cdots, w_n) = \sum_{i=1}^{n} w_i$,
  if $n = 0$ then $\Theta_{sum}() = 0$.
- $\Theta_{sum,\alpha}(w_1, \cdots, w_n) = \left(\sum_{i=1}^{n} (w_i)^{\alpha}\right)^{\frac{1}{\alpha}}$ s.t. $\alpha \geq 1$,
  if $n = 0$ then $\Theta_{sum,\alpha}() = 0$.
- $\Theta_{psum}(w_1, \cdots, w_n) = w_1 \ominus \cdots \ominus w_n$, where $w_1 \ominus w_2 = w_1 + w_2 - w_1 \cdot w_2$, if $n = 0$ then $\Theta_{psum}() = 0$ and if $n = 1$ then $\Theta_{psum}(w) = w$.

Temporal Parametric Semantics (TPS)

TPS	Temporal MAP Inferences	Interpretations Strength
$\langle \Delta_{\mathtt{tCon}}, \sigma_{id}, \Theta_{sum} \rangle$	$\{\{F_1, F_2, F_3, F_5, F_6, GR_1, GR_2\}\}$	5
$\langle \Delta_{\mathtt{pCon}}, \sigma_{id}, \Theta_{sum} \rangle$	$\{\{F_1, F_2, F_3, F_5, F_6, GR_1, GR_2\}\}$	5
$\langle \Delta_{\mathtt{tInc}}, \sigma_{id}, \Theta_{sum} \rangle$	$\{\{F_1, F_2, F_3, F_4, F_5, F_6, GR_1\}\}$	5.1
$\langle \Delta_{\mathtt{tCon}}, \sigma_{id}, \Theta_{sum,3} \rangle$	$\{\{F_1, F_2, F_3, F_4, GR_1\}\}$	1.545
$\langle \Delta_{\mathtt{pCon}}, \sigma_{id}, \Theta_{sum,3} \rangle$	$\{\{F_1, F_2, F_3, F_4, F_5, GR_1\}\}$	1.548
$\langle \Delta_{\mathtt{tInc}}, \sigma_{id}, \Theta_{sum,3} \rangle$	$\{\{F_1, F_2, F_3, F_4, F_5, F_6, GR_1\}\}$	1.557
$\langle \Delta_{\mathtt{tCon}}, \sigma_{rule}, \Theta_{sum} \rangle$	$\{\{F_1, F_2, F_3, F_4, GR_1\}\}$	4.4
$\langle \Delta_{\mathtt{pCon}}, \sigma_{rule}, \Theta_{sum} \rangle$	$\{\{F_1, F_2, F_3, F_4, F_5, GR_1\}\}$	4.7
$\langle \Delta_{\mathtt{tInc}}, \sigma_{rule}, \Theta_{sum} \rangle$	$\{\{F_1, F_2, F_3, F_4, F_5, F_6, GR_1\}\}$	5.1
$\langle \Delta_{\mathtt{tCon}}, \sigma_{rule}, \Theta_{sum,3} \rangle$	$\{\{F_1, F_2, F_3, F_4, GR_1\}\}$	1.545
$\langle \Delta_{\mathtt{pCon}}, \sigma_{rule}, \Theta_{sum,3} \rangle$	$\{\{F_1, F_2, F_3, F_4, F_5, GR_1\}\}$	1.548
$\langle \Delta_{\mathtt{tInc}}, \sigma_{rule}, \Theta_{sum,3} \rangle$	$\{\{F_1, F_2, F_3, F_4, F_5, F_6, GR_1\}\}$	1.557

Table: Temporal Parametric Semantics

Conclusion and Perspectives

Conclusion

- Extension of TMLN with temporal predicates and uncertain rules
- Study of different temporal consistencies
- Proposition of parameterizable semantics for MAP inferences

Perspectives

- Implementing this model with NEO4J and semantics in Python
- Add some uncertainty to the time interval
- Extending the model to the argumentation framework