

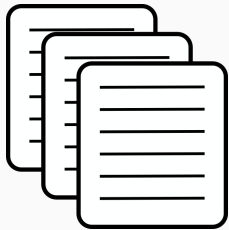
# **DIG Seminar April 2021**

Ranked Enumeration of MSO Logic on Words

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Louis Jachiet joint with Pierre Bourhis, Alejandro Grez and Cristian Riveros

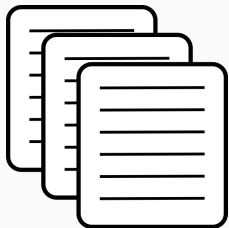
# The problem



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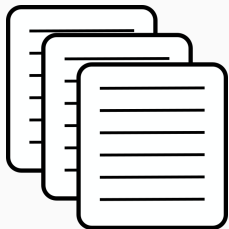


# The problem



Attribute <sub>1</sub>	Attribute <sub>2</sub>	Attribute <sub>3</sub>
<i>A</i> <sub>1</sub>	<i>B</i> <sub>1</sub>	<i>C</i> <sub>1</sub>
<i>A</i> <sub>2</sub>	<i>B</i> <sub>2</sub>	<i>C</i> <sub>2</sub>
<i>A</i> <sub>3</sub>	<i>B</i> <sub>3</sub>	<i>C</i> <sub>3</sub>
<i>A</i> <sub>4</sub>	<i>B</i> <sub>4</sub>	<i>C</i> <sub>4</sub>
...		

# The problem



Attribute <sub>1</sub>	Attribute <sub>2</sub>	Attribute <sub>3</sub>
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<i>A</i> <sub>3</sub>	<i>B</i> <sub>3</sub>	<i>C</i> <sub>3</sub>
<i>A</i> <sub>4</sub>	<i>B</i> <sub>4</sub>	<i>C</i> <sub>4</sub>
...		

Information Extraction

# Enumeration algorithms

We use the **enumeration** framework where the computation is split into two phases:

- **The preprocessing phase**

where we read the input and build some sort of index

- **The enumeration phase**

where we actually output the solutions one-by-one

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PREPROCESSING

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ENUMERATION

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We use the **enumeration** framework where the computation is split into two phases:

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**PREPROCESSING  
TIME**

- **The enumeration phase**

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**DELAY**



# Enumeration algorithms

We use the **enumeration** framework where the computation is split into two phases:

- **The preprocessing phase**

where we read the input and build some sort of index

**PREPROCESSING  
TIME**

Goal: Input Linear

- **The enumeration phase**

where we actually output the solutions one-by-one

**DELAY**

Goal: Constant

## Example

*Lorem ipsum dolor sit amet, consectetur adipiscing elit 2020-03-14 aliquam. Mauris suscipit, suscipit est, sit amet laoreet lorem nisl non lorem. Nam quis eleifend mauris. Donec volutpat turpis rutrum magna pellentesque, sed sodales leo blandit. Proin dolor purus, fringilla non nisl sed, fermentum tincidunt mi. In placerat 2012-01-03 nulla eget leo laoreet, vel gravida lorem finibus. Integer nec imperdiet mauris. Mauris a porttitor ipsum. Nulla facilisi. Phasellus scelerisque hendrerit velit, at euismod nulla 1952-05-12 consequat et. 2015-11-05 massa, id cursus.*

### Extract dates

date

---

## Example

*Lorem ipsum dolor sit amet, consectetur adipiscing elit 2020-03-14 aliquam. Mauris suscipit, suscipit est, sit amet laoreet lorem nisl non lorem. Nam quis eleifend mauris. Donec volutpat turpis rutrum magna pellentesque, sed sodales leo blandit. Proin dolor purus, fringilla non nisl sed, fermentum tincidunt mi. In placerat 2012-01-03 nulla eget leo laoreet, vel gravida lorem finibus. Integer nec imperdiet mauris. Mauris a porttitor ipsum. Nulla facilisi. Phasellus scelerisque hendrerit velit, at euismod nulla 1952-05-12 consequat et. 2015-11-05 massa, id cursus.*

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date
2020-03-14

## Example

*Lorem ipsum dolor sit amet, consectetur adipiscing elit 2020-03-14 aliquam. Mauris suscipit, suscipit est, sit amet laoreet lorem nisl non lorem. Nam quis eleifend mauris. Donec volutpat turpis rutrum magna pellentesque, sed sodales leo blandit. Proin dolor purus, fringilla non nisl sed, fermentum tincidunt mi. In placerat 2012-01-03 nulla eget leo laoreet, vel gravida lorem finibus. Integer nec imperdiet mauris. Mauris a porttitor ipsum. Nulla facilisi. Phasellus scelerisque hendrerit velit, at euismod nulla 1952-05-12 consequat et. 2015-11-05 massa, id cursus.*

### Extract dates

<u>date</u>
2020-03-14
2012-01-03

## Example

*Lorem ipsum dolor sit amet, consectetur adipiscing elit 2020-03-14 aliquam. Mauris suscipit, suscipit est, sit amet laoreet lorem nisl non lorem. Nam quis eleifend mauris. Donec volutpat turpis rutrum magna pellentesque, sed sodales leo blandit. Proin dolor purus, fringilla non nisl sed, fermentum tincidunt mi. In placerat 2012-01-03 nulla eget leo laoreet, vel gravida lorem finibus. Integer nec imperdiet mauris. Mauris a porttitor ipsum. Nulla facilisi. Phasellus scelerisque hendrerit velit, at euismod nulla 1952-05-12 consequat et. 2015-11-05 massa, id cursus.*

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## Example

*Lorem ipsum dolor sit amet, consectetur adipiscing elit 2020-03-14 aliquam. Mauris suscipit, suscipit est, sit amet laoreet lorem nisl non lorem. Nam quis eleifend mauris. Donec volutpat turpis rutrum magna pellentesque, sed sodales leo blandit. Proin dolor purus, fringilla non nisl sed, fermentum tincidunt mi. In placerat 2012-01-03 nulla eget leo laoreet, vel gravida lorem finibus. Integer nec imperdiet mauris. Mauris a porttitor ipsum. Nulla facilisi. Phasellus scelerisque hendrerit velit, at euismod nulla 1952-05-12 consequat et. 2015-11-05 massa, id cursus.*

### Extract dates

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2012-01-03
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## Example

*Mr. Emmanuel Macron was born on the 1977-12-21 and is the President of France since the 2017-05-14. Ms. Angela Merkel was born on the 1954-07-17 and she is the head of Germany since the 2005-11-22.*

**Extract pairs people-date appearing in the same sentence.**



## Example

Mr. *Emmanuel Macron* was born on the *1977-12-21* and is the President of France since the *2017-05-14*. Ms. *Angela Merkel* was born on the *1954-07-17* and she is the head of Germany since the *2005-11-22*.

**Extract pairs people-date appearing in the same sentence.**

person	date
Emmanuel Macron	1977-12-21



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Emmanuel Macron	2017-05-14
Angela Merkel	1954-07-17

## Example

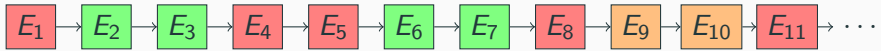
*Mr. Emmanuel Macron was born on the 1977-12-21 and is the President of France since the 2017-05-14. Ms. **Angela Merkel** was born on the 1954-07-17 and she is the head of Germany since the **2005-11-22**.*

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# Another problem

## A stream of events



## Another problem

### A stream of events

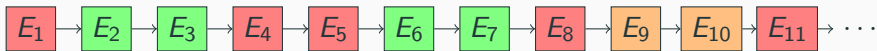


Pairs of red events with no green in-between:



## Another problem

### A stream of events

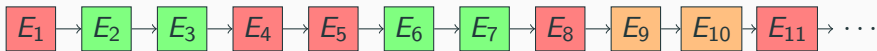


Pairs of red events with no green in-between:

Start	End
$E_4$	$E_5$

## Another problem

### A stream of events



Pairs of red events with no green in-between:

Start	End
$E_4$	$E_5$
$E_8$	$E_{11}$

## Linearly ordered data

- Text
- Streams
- ...



## Linearly ordered data

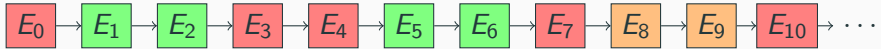
- Text
- Streams
- ...

## Monadic Second Order logic

Basically “regular automaton logic”

# Another problem

## A stream of events

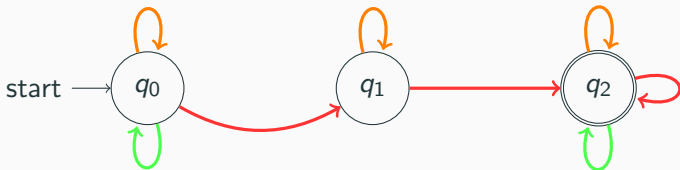


## Another problem

### A stream of events



Pairs  $(x, y)$  of red events with no green in-between:

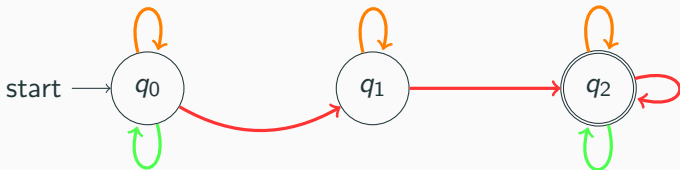


## Another problem

### A stream of events



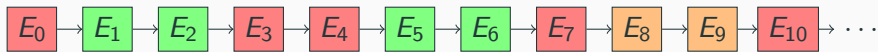
### Pairs $(x, y)$ of red events with no green in-between:



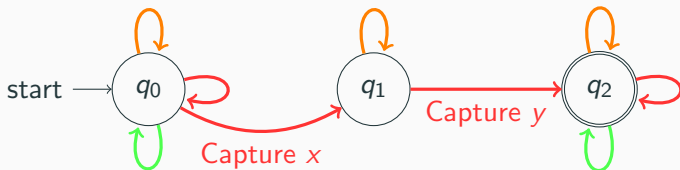
This **accepts** streams but what does it **captures**?

## Another problem

### A stream of events



Pairs  $(x, y)$  of red events with no green in-between:



## Some formalism on automata

### Run

A **run** of an automaton  $\mathcal{A}$  on a word  $w_0 \dots w_{n-1}$  is a sequence  $q_0 \dots q_n$  such that:

- $q_i \xrightarrow{w_i} q_{i+1}$  is a valid transition of  $\mathcal{A}$
- $q_0$  is the initial state of  $\mathcal{A}$
- $q_n$  is a final state of  $\mathcal{A}$

# Some formalism on automata

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## Trace of a run

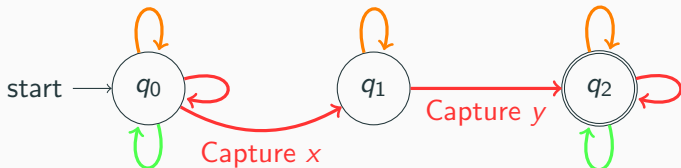
Each transition of  $\mathcal{A}$  has an output. The **trace** of a run  $q_0 \dots q_n$  is  $\{(i, out(q_i, w_i, q_{i+1}) \mid out(q_i, w_i, q_{i+1}) \neq \emptyset\}$ .

## Unambiguous automaton (sort of determinism)

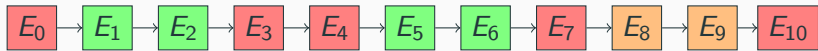
An automaton is **unambiguous** if we cannot have two different runs with the same trace.

# Running an automaton

## Automaton

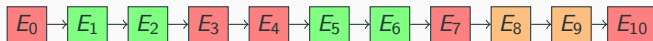
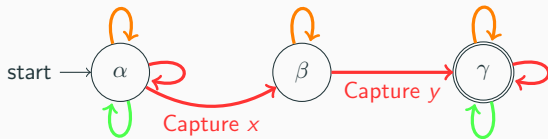


## Run and traces

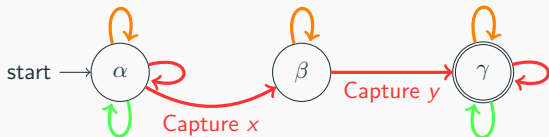




# Product word - automaton



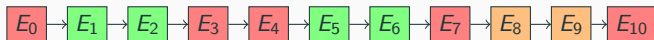
# Product word - automaton



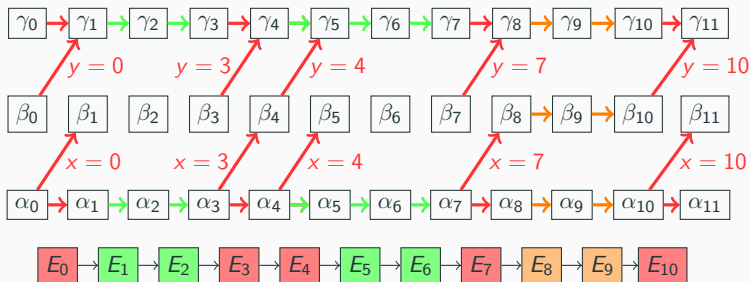
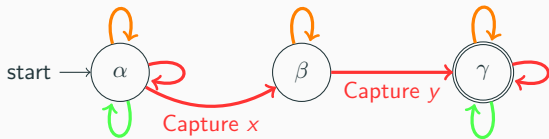
$\gamma_0$   $\gamma_1$   $\gamma_2$   $\gamma_3$   $\gamma_4$   $\gamma_5$   $\gamma_6$   $\gamma_7$   $\gamma_8$   $\gamma_9$   $\gamma_{10}$   $\gamma_{11}$

$\beta_0$   $\beta_1$   $\beta_2$   $\beta_3$   $\beta_4$   $\beta_5$   $\beta_6$   $\beta_7$   $\beta_8$   $\beta_9$   $\beta_{10}$   $\beta_{11}$

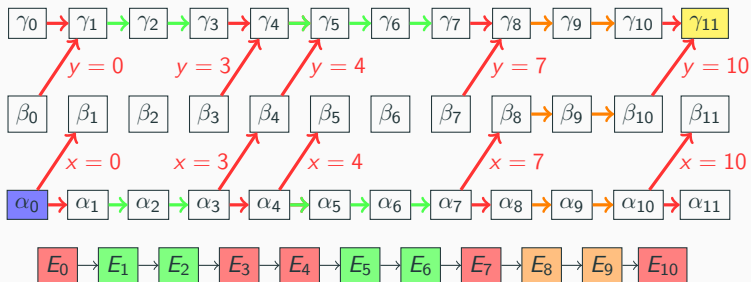
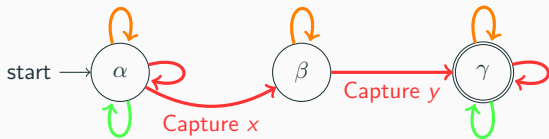
$\alpha_0$   $\alpha_1$   $\alpha_2$   $\alpha_3$   $\alpha_4$   $\alpha_5$   $\alpha_6$   $\alpha_7$   $\alpha_8$   $\alpha_9$   $\alpha_{10}$   $\alpha_{11}$



# Product word - automaton



# Product word - automaton



# Enumeration of traces

boils down to

enumeration of

paths in a DAG

Enumeration order depends on  
the **algorithm used**  
and not on the  
**relevance** of results

Cost MSO and  
Ranked Enumeration of MSO  
Logic on Words

### **Idea**

Each solution comes with a weight



## Idea

Each solution comes with a weight

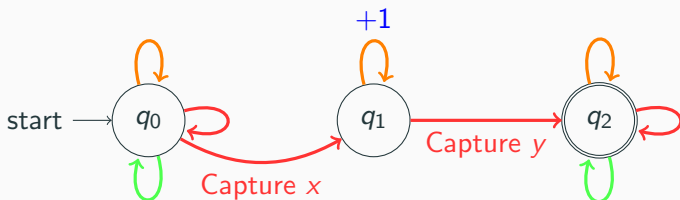
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## Example

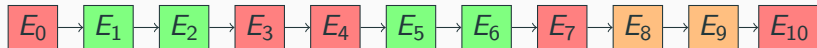
$$\text{Weight}(x, y) = |\{z \mid x < z < y\}|$$

# Cost MSO Logic on Words

**Weighted Automaton with**  $Weight(x, y) = |\{z \mid x < z < y\}|$



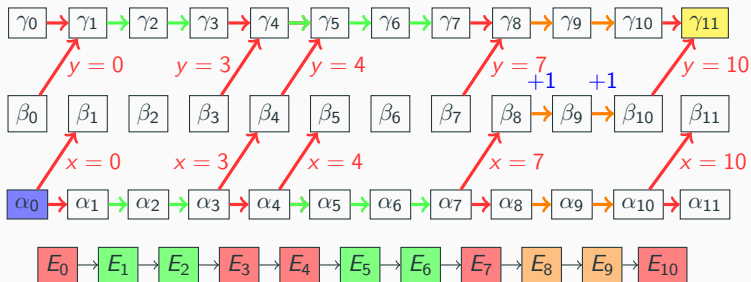
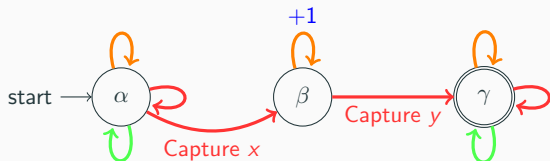
**Run and traces**



**Trace of a run**

The **trace** of a run  $q_0 \dots q_n$  has weight  $\sum w(q_i, w_i, q_{i+1})$ .

# Product word - automaton



## The Heap of Words datastructure

We suppose that we have a datastructure capable of storing **words with priorities** equipped with the following operations

- `isEmpty`

# The Heap of Words datastructure

We suppose that we have a datastructure capable of storing **words with priorities** equipped with the following operations

- ISEMPTY
- ADD

# The Heap of Words datastructure

We suppose that we have a datastructure capable of storing **words with priorities** equipped with the following operations

- ISEMPTY
- ADD
- UNION

# The Heap of Words datastructure

We suppose that we have a datastructure capable of storing **words with priorities** equipped with the following operations

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- ADD
- UNION
- FINDMIN

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We suppose that we have a datastructure capable of storing **words with priorities** equipped with the following operations

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- FINDMIN
- DELETEMIN



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- DELETEMIN
- INCREASEBY

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- DELETEMIN
- INCREASEBY
- EXTENDBY

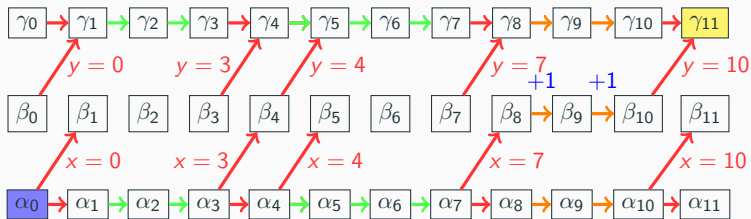
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- INCREASEBY
- EXTENDBY

We also ask that the datastructure is **persistent**.

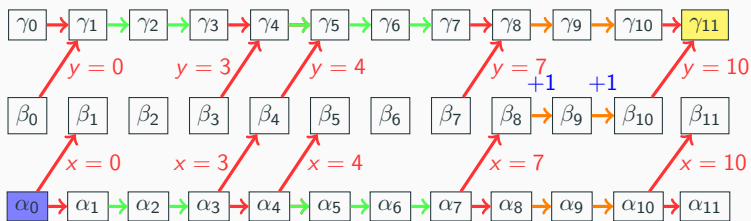
# Product word - automaton



**For a node  $n$**

$$\text{Paths}(n) = \text{Union}(\text{Paths}(n, n'))$$

## Product word - automaton



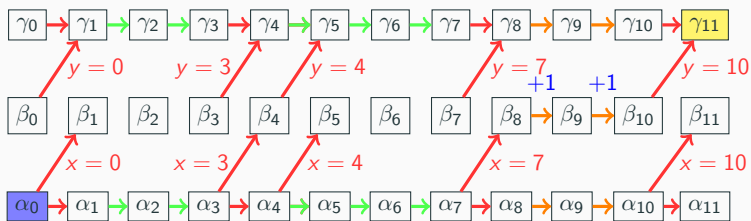
**For a node  $n$**

$$\text{Paths}(n) = \text{Union}(\text{Paths}(n, n'))$$

**For an edge  $(u, v)$  with a weight  $w$**

$$\text{Paths}((u, v)) = \text{INCREASEBY}(\text{Paths}(v), w)$$

## Product word - automaton



**For a node  $n$**

$$\text{Paths}(n) = \text{Union}(\text{Paths}(n, n'))$$

**For an edge  $(u, v)$  with a weight  $w$**

$$\text{Paths}((u, v)) = \text{INCREASEBY}(\text{Paths}(v), w)$$

**For an edge  $(u, v)$  with a label  $l$  and a weight  $w$**

$$\text{Paths}((u, v)) = \text{INCREASEBY}(\text{EXTENDBY}(\text{Paths}(v), l), w)$$

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- UNION
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- INCREASEBY
- EXTENDBY

We also asks that the datastructure is **persistent**.

Questions?

# The Brodal Queue

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# The Heap of Words datastructure

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