Beyond NP Revolution

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Artificial Intelligence and Logic

Turing, 1950: "Opinions may vary as to the complexity which is suitable in the child machine. One might try to make it as simple as possible consistent with the general principles. Alternatively one might have a complete system of logical inference "built in". In the latter case the store would be largely occupied with definitions and propositions. The propositions would have various kinds of status, e.g., well-established facts, conjectures, mathematically proved theorems, statements given by an authority,...'

Aristotle's Syllogisms

- All men are mortal
- Socrates is a man

Socrates is a mortal

Boole's Symbolic Logic

Boole's insight: Aristotle's syllogisms are about *classes* of objects, which can be treated *algebraically*.

> *"If an adjective, as 'good', is employed as a term of description, let us represent by a letter, as y, all things to which the description 'good' is applicable, i.e., 'all good things', or the class of 'good things'. Let it further be agreed that by the combination xy shall be represented that class of things to which the name or description represented by x and y are simultaneously applicable. Thus, if x alone stands for 'white' things and y for 'sheep', let xy stand for 'white sheep'.*

Boolean Satisfiability

Boolean Satisfiability (SAT); Given a Boolean expression, using "and" ($\wedge$) "or", ($\vee$) and "not" ($\neg$), *is there a satisfying solution* (an assignment of 0's and 1's to the variables that makes the expression equal 1)?
Example:

$$(\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \wedge (x_3 \vee x_1 \vee x_4)$$

Solution: $x_1 = 0$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$

History:

- William Stanley Jevons, 1835-1882: "I have given much attention, therefore, to lessening both the manual and mental labour of the process, and I shall describe several devices which may be adopted for saving trouble and risk of mistake."
- Ernst Schröder, 1841-1902: "Getting a handle on the consequences of any premises, or at least the fastest method for obtaining these consequences, seems to me to be one of the noblest, if not the ultimate goal of mathematics and logic."

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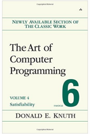
- Clay Institute, 2000: $1M Award!

Algorithmic Boolean Reasoning: Early History

- Davis and Putnam, 1958: "Computational Methods in The Propositional calculus", unpublished report to the NSA
- Davis and Putnam, JACM 1960: "A Computing procedure for quantification theory"
- Davis, Logemman, and Loveland, CACM 1962: "A machine program for theorem proving"
- Marques-Silva and Sakallah 1996, Zhang et al. 2001, Een and Sorensson 2003, Simon and Audemard 2009, Liang et al 2016
  CDCL = conflict-driven clause learning
  - Smart but cheap branching heuristics
  - Quick detection of unit clauses
  - Conflict Driven Clause Learning
  - Restarts

The Tale of Triumph of SAT Solvers

Modern SAT solvers are able to deal routinely with practical problems that involve many thousands of variables, although such problems were regarded as hopeless just a few years ago. (Donald Knuth, 2016)



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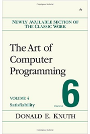

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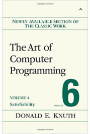

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Now that SAT is "easy", it is time to look beyond satisfiability

Constrained Counting and Sampling

- Given
  - Boolean variables $X_1, X_2, \cdots X_n$
  - Formula F over $X_1, X_2, \cdots X_n$

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- $W(F) = \Sigma_{y \in \mathsf{Sol}(F)} W(y)$
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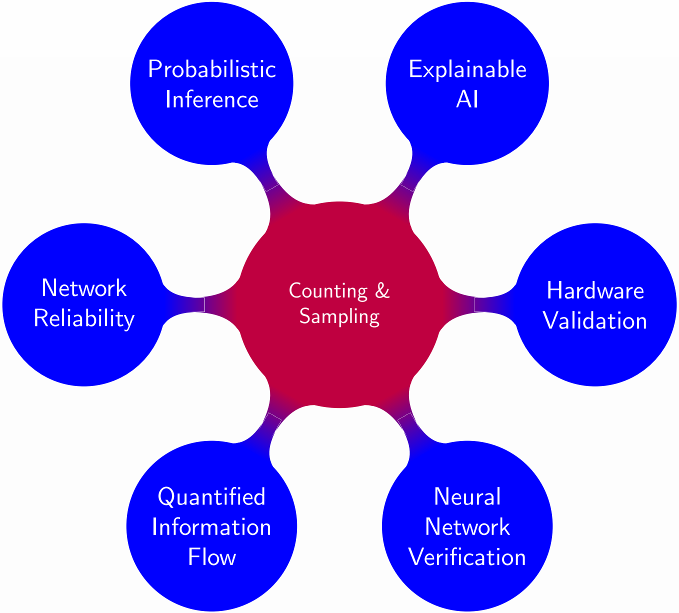
Applications across Computer Science



Today's Menu

Network Reliability

Probabilistic Inference

Hardware Validation

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Constrained Counting

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Hashing Framework

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Hashing Framework

Can we reliably predict the effect of natural disasters on critical infrastructure such as power grids?

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Can we predict likelihood of a region facing blackout?

Reliability of Critical Infrastructure Networks

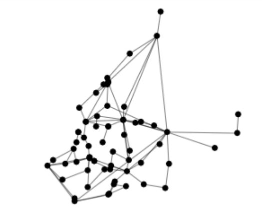

Figure: Plantersville, SC

- $G = (V, E)$; source node: s and terminal node t
- failure probability $g : E \to [0, 1]$
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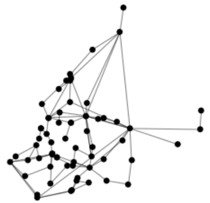

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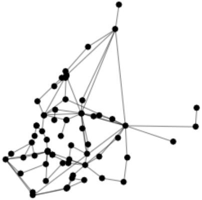

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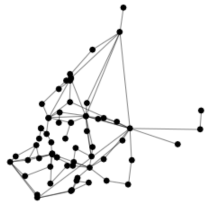

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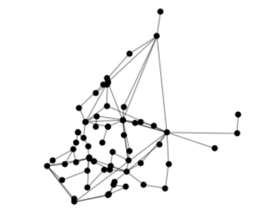

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(DMPV, AAAI 17, ICASP13 2019)

Probabilistic Models

Patient	Cough	Smoker	Asthma
Alice	1	0	0
Bob	0	0	1
Randee	1	0	0
Tova	1	1	1
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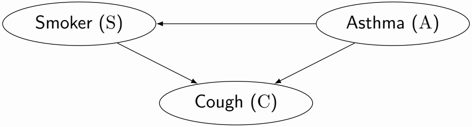

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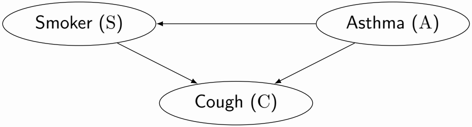

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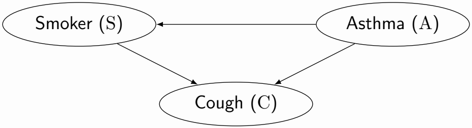

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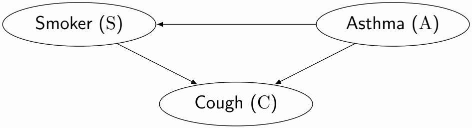

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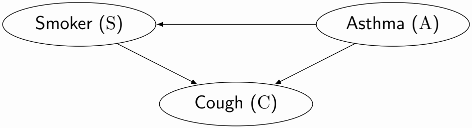

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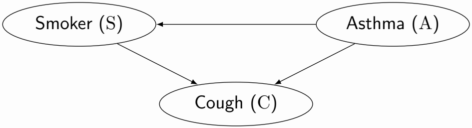

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Constrained Counting

(Roth, 1996)

Prior Work

Strong guarantees but poor scalability

- Exact counters (Birnbaum and Lozinskii 1999, Jr. and Schrag 1997, Sang et al. 2004, Thurley 2006, Lagniez and Marquis 2014-18)
- Hashing-based approach (Stockmeyer 1983, Jerrum Valiant and Vazirani 1986)

Weak guarantees but impressive scalability

- Bounding counters (Gomes et al. 2007,Kroc, Sabharwal, and Selman 2008, Gomes, Sabharwal, and Selman 2006, Kroc, Sabharwal, and Selman 2008)
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How to bridge this gap between theory and practice?

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- ApproxCount($F, W, \varepsilon, \delta$): Compute C such that

$$\Pr[\frac{W(F)}{1+\varepsilon} \leq C \leq W(F)(1+\varepsilon)] \geq 1-\delta$$

From Weighted to Unweighted Counting

Boolean Formula F and weight function $W : \{0,1\}^n \to \mathbb{Q}^{\geq 0}$

Boolean Formula F'

$$W(F) = c(W) \times |\mathsf{Sol}(F')|$$

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How do we estimate $|\mathsf{Sol}(F')|$?

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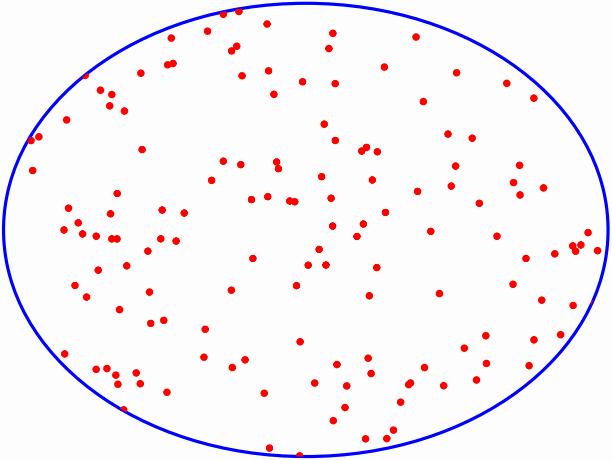
  - Potentially 2^n queries

Can we do with lesser # of SAT queries – $\mathcal{O}(n)$ or $\mathcal{O}(\log n)$?

As Simple as Counting Dots



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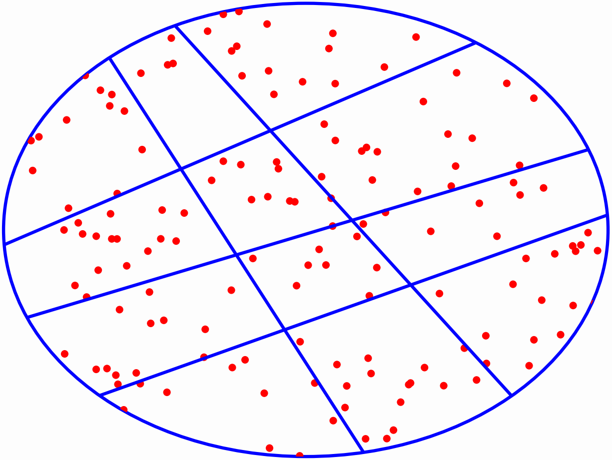
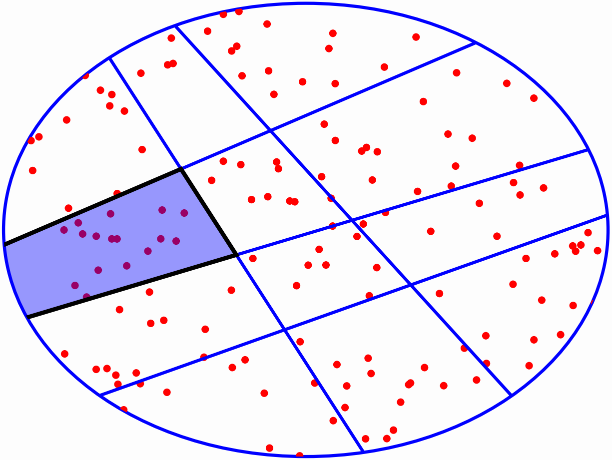


Pick a random cell



Estimate = Number of solutions in a cell × Number of cells

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Challenge 2 How many cells?

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- Designing function h : assignments $\rightarrow$ cells (hashing)
- Solutions in a cell α: $\mathsf{Sol}(F) \cap \{y \mid h(y) = \alpha\}$

Challenges

Challenge 1 How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

- Designing function h : assignments $\rightarrow$ cells (hashing)
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- Choose h randomly from a large family H of hash functions

  Universal Hashing (Carter and Wegman 1977)

2-Universal Hashing

- Let H be family of 2-universal hash functions mapping $\{0,1\}^n$ to $\{0,1\}^m$

$$\forall y_1, y_2 \in \{0,1\}^n, \alpha_1, \alpha_2 \in \{0,1\}^m, h \xleftarrow{R} H$$
$$\Pr[h(y_1) = \alpha_1] = \Pr[h(y_2) = \alpha_2] = \left(\frac{1}{2^m}\right)$$

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- The power of 2-universality
  - Z be the number of solutions in a randomly chosen cell
  - $\mathsf{E}[Z] = \frac{|\mathsf{Sol}(F)|}{2^m}$
  - $\sigma^2[Z] \leq \mathsf{E}[Z]$

2-Universal Hash Functions

- Variables: $X_1, X_2, \cdots X_n$
- To construct $h : \{0,1\}^n \rightarrow \{0,1\}^m$, choose m random XORs
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- Solutions in a cell: $F \wedge Q_1 \cdots \wedge Q_m$
- Performance of state of the art SAT solvers degrade with increase in the size of XORs (SAT Solvers != SAT oracles)

Improved Universal Hash Functions

- Not all variables are required to specify solution space of F
  - $F := X_3 \iff (X_1 \vee X_2)$
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- Formally: if I is independent support, then $\forall \sigma_1, \sigma_2 \in \mathsf{Sol}(F)$, if σ_1 and σ_2 agree on I then $\sigma_1 = \sigma_2$
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Algorithmic procedure to determine I?

- FP^{NP} procedure via reduction to Minimal Unsatisfiable Subset
- Two orders of magnitude runtime improvement
  (IMMV CP15, Best Student Paper) (IMMV Constraints16, Invited Paper)

Challenges

Challenge 1 How to partition into roughly equal small cells of solutions without knowing the distribution of solutions?

- Independent Support-based 2-Universal Hash Functions

Challenge 2 How many cells?

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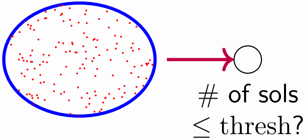
  - Check for every $m = 0, 1, \cdots n$ if the number of solutions $\leq \mathrm{thresh}$

ApproxMC(F, ε, δ)



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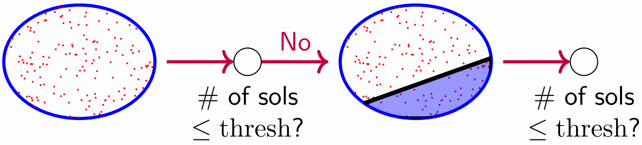

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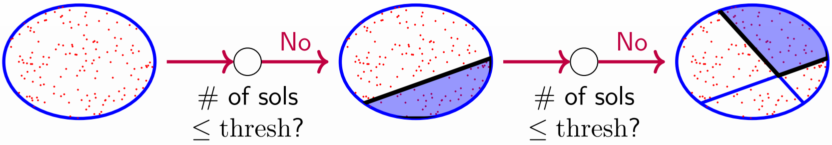

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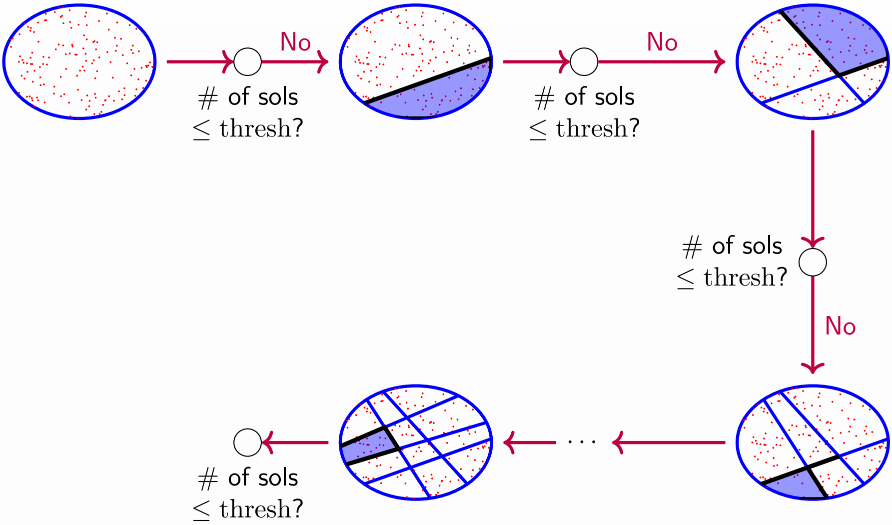

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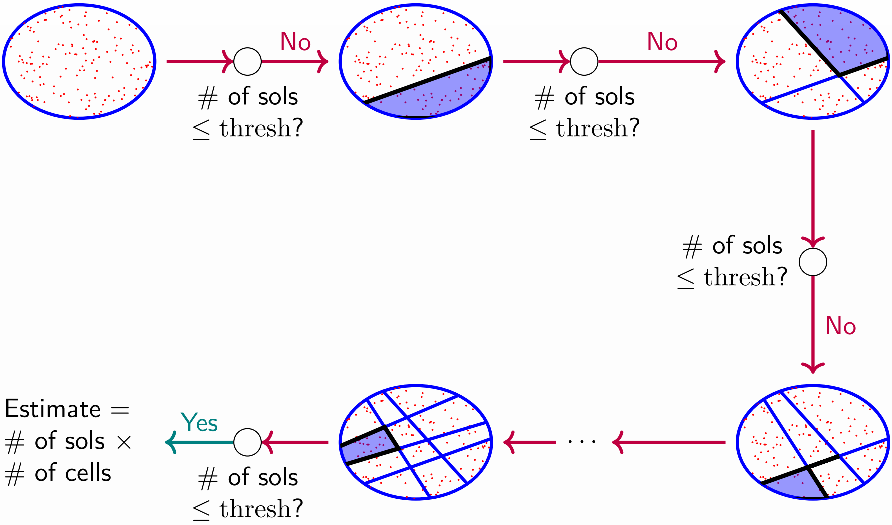

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- We want to partition into 2^{m^*} cells such that $2^{m^*} = \frac{|\mathsf{Sol}(F)|}{\mathrm{thresh}}$
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- Stop at the first m where Query m returns YES and return estimate as $\#(F \wedge Q_1 \wedge Q_2 \cdots \wedge Q_m) \times 2^m$
- Observation: $\#(F \wedge Q_1 \cdots \wedge Q_i \wedge Q_{i+1}) \leq \#(F \wedge Q_1 \cdots \wedge Q_i)$
  - If Query i returns YES, then Query $i+1$ must return YES

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(CMV, IJCAI16)

ApproxMC(F, ε, δ)

Theorem (Correctness)

$$\Pr\left[\frac{|\mathsf{Sol}(F)|}{1+\varepsilon} \le ApproxMC(F,\varepsilon,\delta) \le |\mathsf{Sol}(F)|(1+\varepsilon)\right] \ge 1-\delta$$

Theorem (Complexity)

ApproxMC(F, ε, δ) *makes* $\mathcal{O}(\frac{\log n \log(\frac{1}{\delta})}{\varepsilon^2})$ *calls to SAT oracle.*

- *Prior work required* $\mathcal{O}(\frac{\boldsymbol{n \log n} \log(\frac{1}{\delta})}{\varepsilon})$ *calls to SAT oracle (Stockmeyer 1983)*

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Theorem (FPRAS for DNF; (MSV, FSTTCS-17; CP-18, IJCAI-29(Invited Paper)))

If F is a DNF formula, then ApproxMC is FPRAS – fundamentally different from the only other known FPRAS for DNF (Karp, Luby 1983)

Reliability of Critical Infrastructure Networks

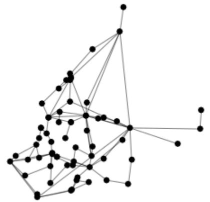
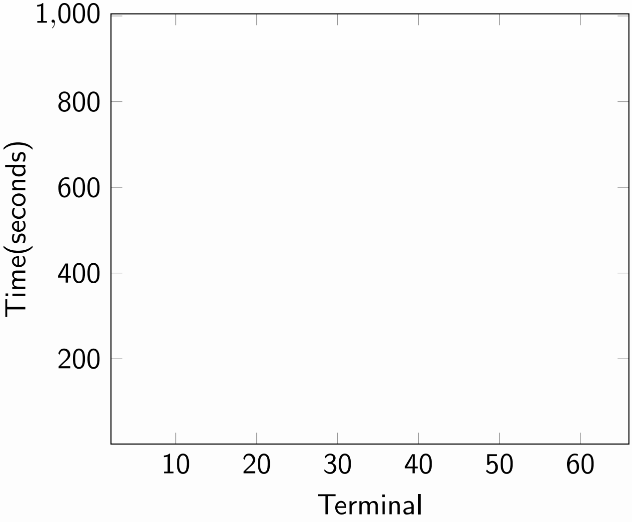


Figure: Plantersville, SC

- $G = (V, E)$; source node: s
- Compute Pr[t is disconnected]?

Timeout = 1000 seconds



(DMPV, AAAI17)

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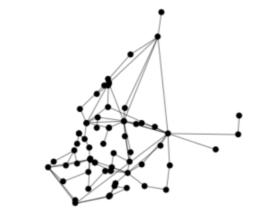

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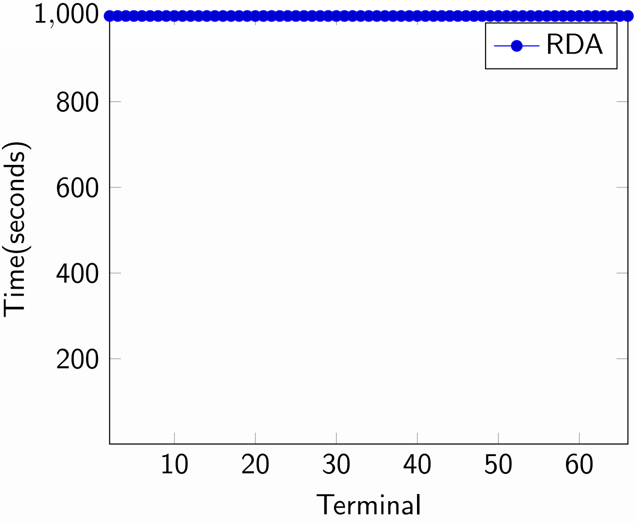

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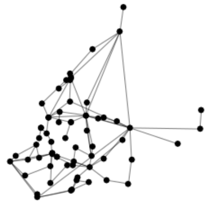

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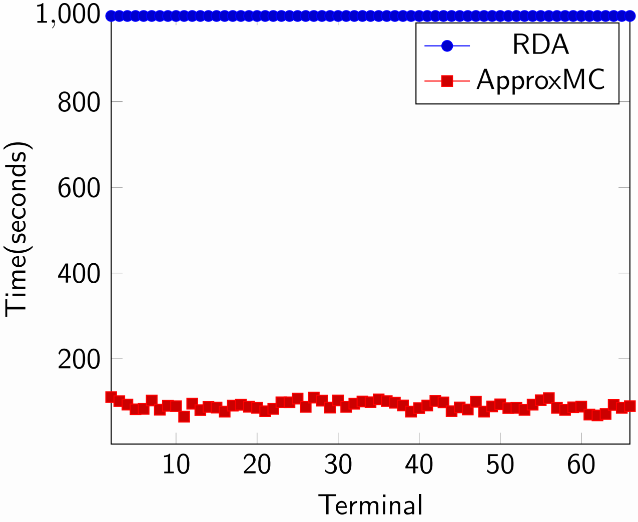

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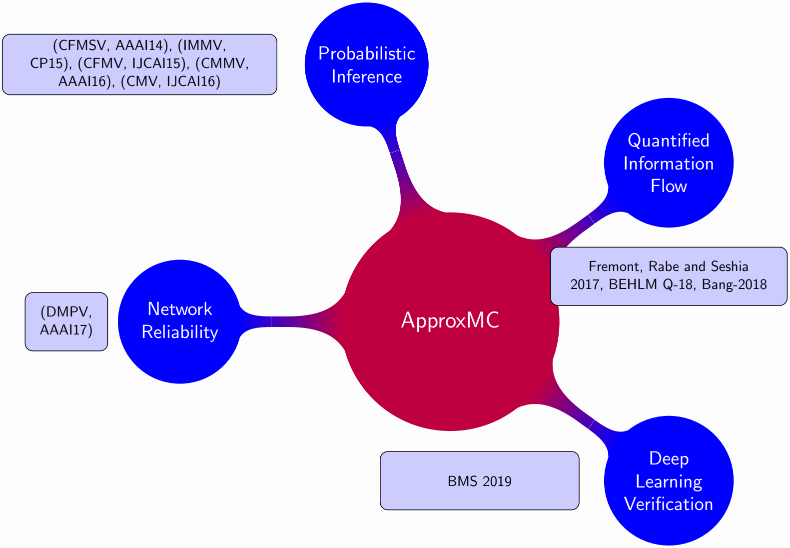
Beyond Network Reliability

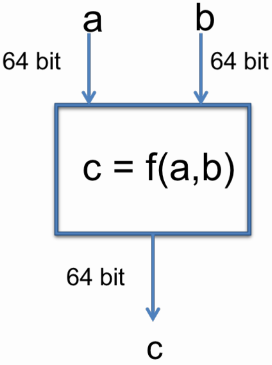


Network Reliability

Probabilistic Inference

Constrained Counting

Hardware Validation



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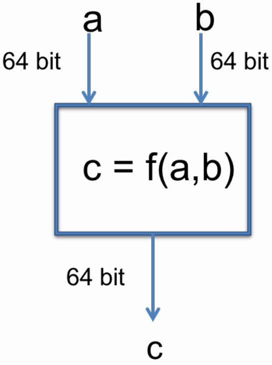

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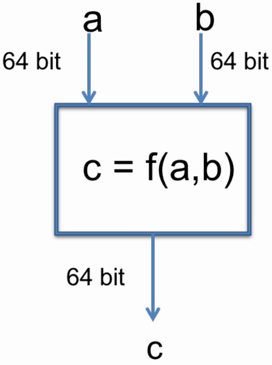

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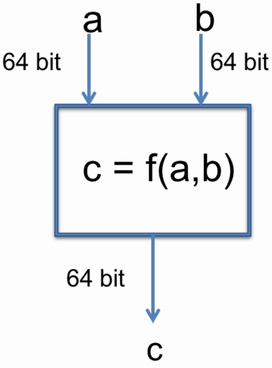
  - 2^{128} combinations for a toy circuit
- Use constraints to represent *interesting* verification scenarios

Constrained-Random Simulation



Constraints

- Designers:
  - $a +_{64} 11 * 32b = 12$
  - $a <_{64} (b >> 4)$
- Past Experience:
  - $40 <_{64} 34 + a <_{64} 5050$
  - $120 <_{64} b <_{64} 230$
- Users:
  - $232 * 32a +_{64} b! = 1100$
  - $1020 <_{64} (b/_{64}2) +_{64} a <_{64} 2200$

Test vectors: random solutions of constraints

Constrained Sampling

- Given:
  - Set of Constraints F over variables $X_1, X_2, \cdots X_n$
- Uniform Sampler

$$\forall y \in \mathsf{Sol}(F), \mathsf{Pr}[\text{y is output}] = \frac{1}{|\mathsf{Sol}(F)|}$$

- Almost-Uniform Sampler

$$\forall y \in \mathsf{Sol}(F), \frac{1}{(1+\varepsilon)|\mathsf{Sol}(F)|} \leq \mathsf{Pr}[\text{y is output}] \leq \frac{(1+\varepsilon)}{|\mathsf{Sol}(F)|}$$

Prior Work

Strong guarantees but poor scalability

- Polynomial calls to NP oracle (Bellare, Goldreich and Petrank,2000)
- BDD-based techniques (Yuan et al 1999, Yuan et al 2004, Kukula and Shiple 2000)
- Reduction to approximate counting (Jerrum, Valiant and Vazirani 1986)

Weak guarantees but impressive scalability

- Randomization in SAT solvers (Moskewicz 2001, Nadel 2011)
- MCMC-based approaches (Sinclair 1993, Jerrum and Sinclair 1996, Kitchen and Kuehlmann 2007,...)
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How to bridge this gap between theory and practice?

Close Cousins: Counting and Sampling

- Approximate counting and almost-uniform sampling are inter-reducible (Jerrum, Valiant and Vazirani, 1986)

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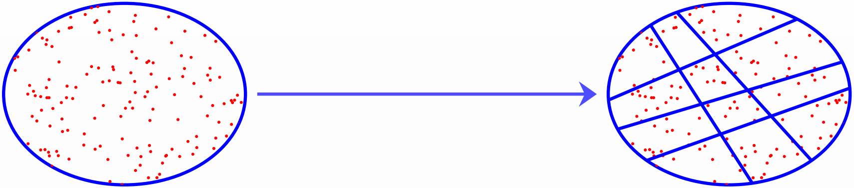
- Approximate counting and almost-uniform sampling are inter-reducible (Jerrum, Valiant and Vazirani, 1986)
- Is the reduction efficient?
  - Almost-uniform sampler (JVV) require linear number of approximate counting calls

Key Ideas



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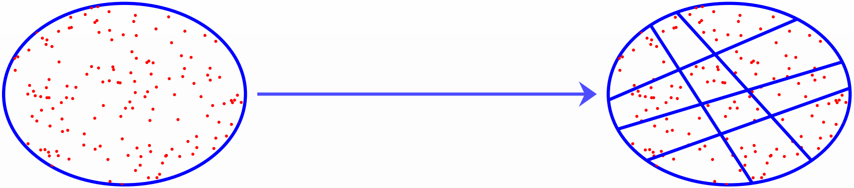

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  - Not just a practical hack required non-trivial proof

(CMV, CAV13)
(CMV, DAC14),
(CFMSV, AAAI14),
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Theoretical Guarantees

Theorem (Almost-Uniformity)

$$\forall y \in \mathsf{Sol}(F),\ \frac{1}{(1+\varepsilon)|\mathsf{Sol}(F)|} \leq \Pr[y \textit{ is output}] \leq \frac{1+\varepsilon}{|\mathsf{Sol}(F)|}$$

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Theorem (Query)

For a formula F over n variables UniGen makes **one call** *to approximate counter*

Theoretical Guarantees

Theorem (Almost-Uniformity)

$$\forall y \in \mathsf{Sol}(F),\ \frac{1}{(1+\varepsilon)|\mathsf{Sol}(F)|} \leq \Pr[y \textit{ is output}] \leq \frac{1+\varepsilon}{|\mathsf{Sol}(F)|}$$

Theorem (Query)

For a formula F over n variables UniGen makes **one call** *to approximate counter*

- *Prior work required* **n** *calls to approximate counter (Jerrum, Valiant and Vazirani, 1986)*

	Relative Runtime
SAT Solver	1
Desired Uniform Generator	10

Experiments over 200+ benchmarks

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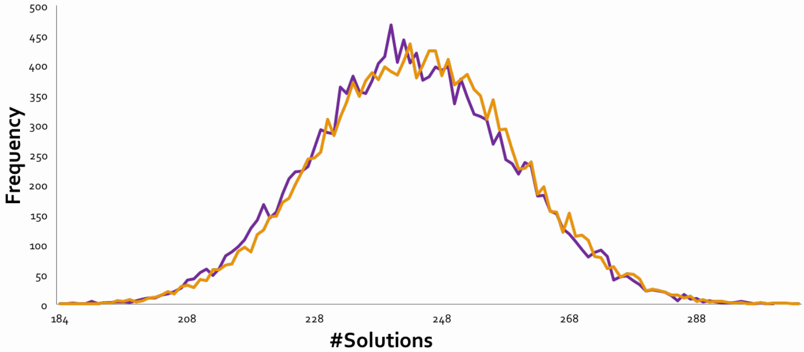
Experiments over 200+ benchmarks

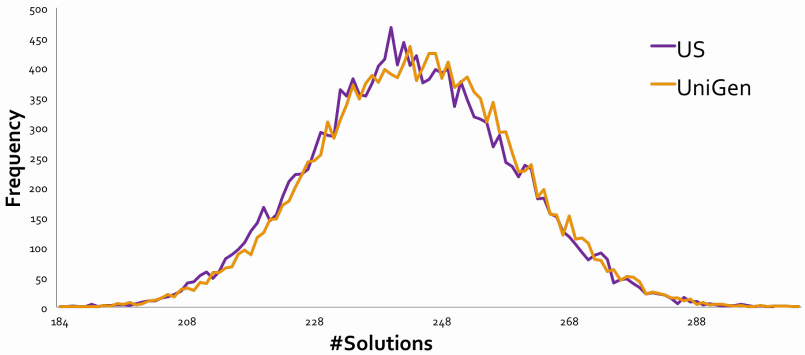
Closer to technical transfer

Quiz Time: Uniformity



- Benchmark: case110.cnf; #var: 287; #clauses: 1263
- Total Runs: 4×10^6; Total Solutions : 16384

Statistically Indistinguishable



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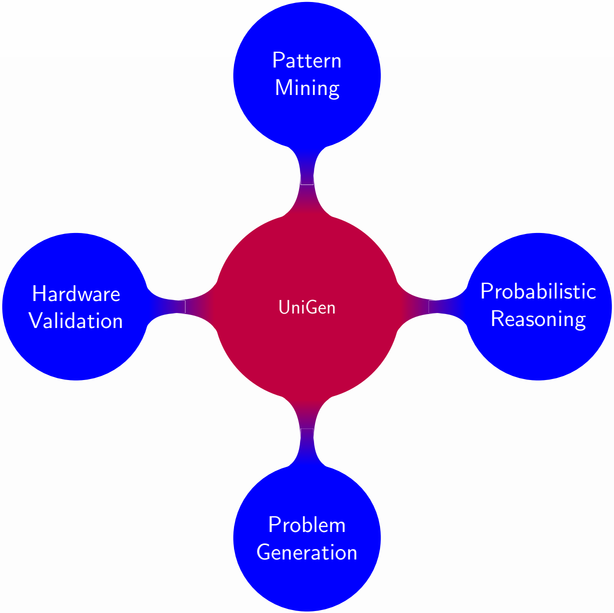
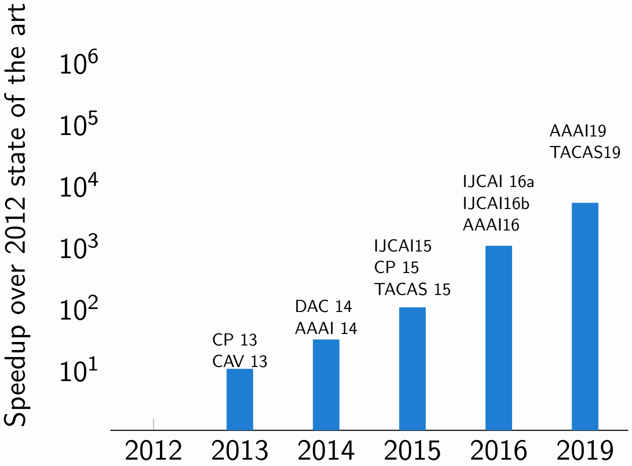
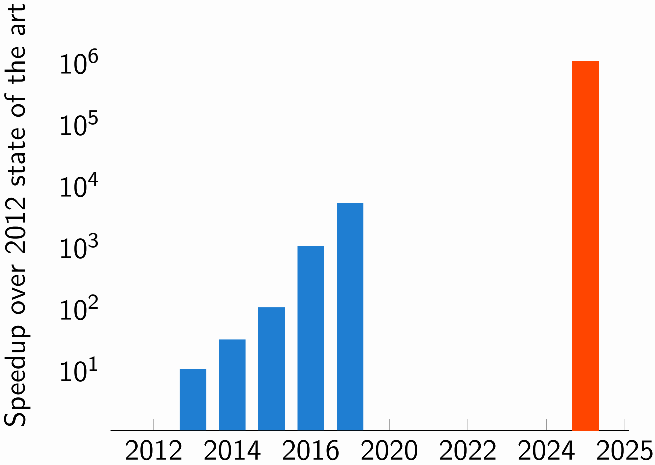
Usages of Open Source Tool: UniGen




Speedup over 2012 state of the art
10^6
10^5
10^4
10^3
10^2
10^1
CP 13
CAV 13
DAC 14
AAAI 14
IJCAI15
CP 15
TACAS 15
IJCAI 16a
IJCAI16b
AAAI16
AAAI19
TACAS19
2012
2013
2014
2015
2016
2019

Mission 2025: Constrained Counting and Sampling Revolution



Requires combinations of ideas from theory, statistics and systems

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- Tighter integration between solvers and algorithms (SM, AAAI19)

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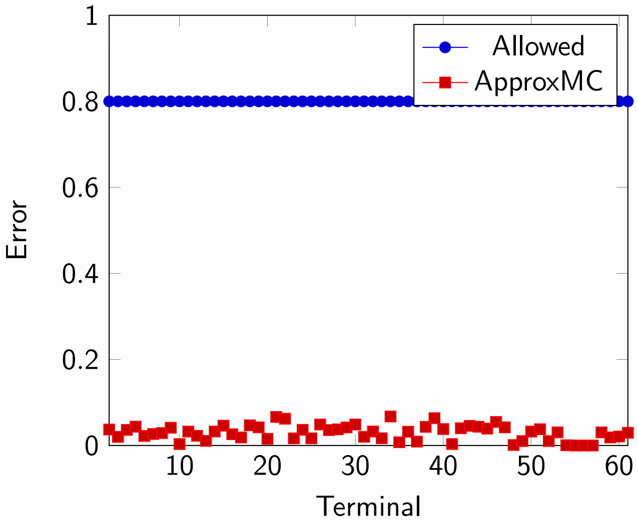
Join us in our mission: Positions for long-term research assistants, PhD students, and postdocs. Visit meelgroup.github.io for details on how to apply.

Part I

Backup

Highly Accurate Estimates



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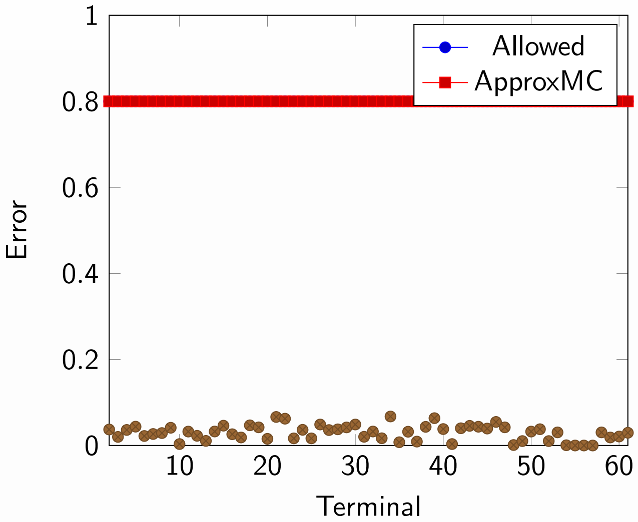
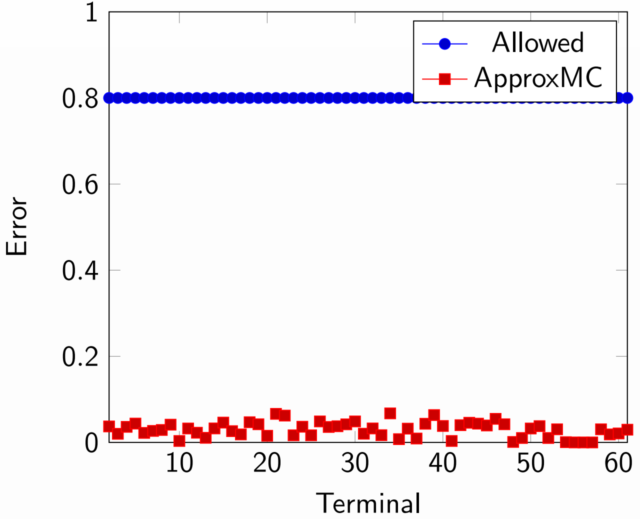


Observed Geometric mean: 0.03

Highly Accurate Estimates



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These results are good

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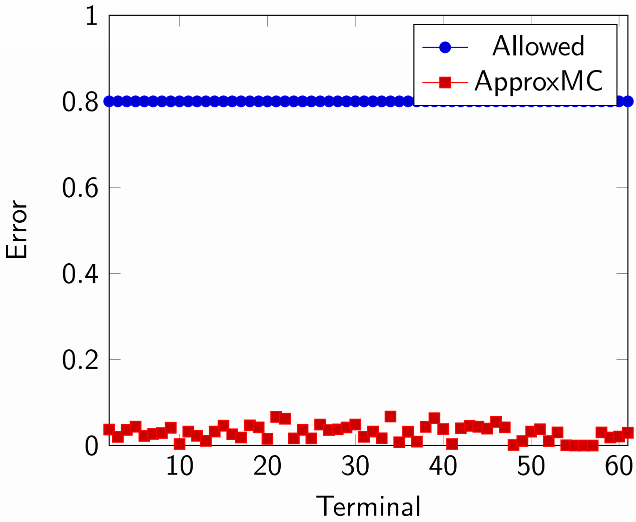

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These results are good problem.

Independent Support

- $I \subseteq X$ is an independent support:
  $\forall \sigma_1, \sigma_2 \in \mathsf{Sol}(\varphi)$, σ_1 and σ_2 agree on I then $\sigma_1 = \sigma_2$

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  where $F(y_1, \cdots y_n) := F(x_1 \rightarrowtail y_1, \cdots x_n \rightarrowtail y_n)$

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- $Q_{F,I} := F(x_1, \cdots x_n) \wedge F(y_1, \cdots y_n) \wedge \bigwedge_{i \mid x_i \in I}(x_i = y_i) \wedge \neg(\bigwedge_i (x_i = y_i))$

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- Lemma: $Q_{F,I}$ is UNSAT if and only if I is independent support

Independent Support

$$H_1 := \{x_1 = y_1\}, H_2 := \{x_2 = y_2\}, \cdots H_n := \{x_n = y_n\}$$

$$\Omega = F(x_1, \cdots x_n) \wedge F(y_1, \cdots y_n) \wedge \neg(\bigwedge_i (x_i = y_i))$$

Lemma

$I = \{x_i\}$ is independent support iif $H^I \wedge \Omega$ is UNSAT where $H^I = \{H_i | x_i \in I\}$

Minimal Unsatisfiable Subset

Given $\Psi = H_1 \wedge H_2 \cdots \wedge H_m \wedge \Omega$

Unsatisfiable Subset Find subset $\{H_{i1}, H_{i2}, \cdots H_{ik}\}$ of $\{H_1, H_2, \cdots H_m\}$ such that $H_{i1} \wedge H_{i2} \wedge H_{ik} \wedge \Omega$ is UNSAT

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Two orders of magnitude improvement in runtime