# Top-k Queries over Uncertain Scores

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- Examples: TripAdvisor
  - collaborative user or crowdsourced collection of information, e.g., user generated ratings and reviews, to recommend travel plans and hotels, vacation rentals and restaurants.

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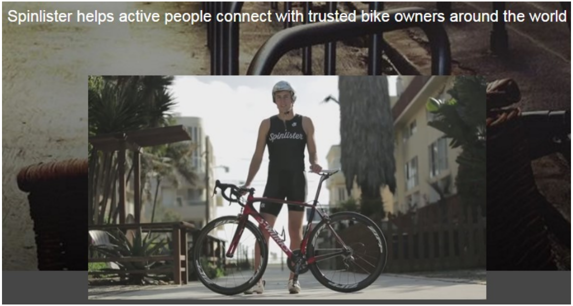

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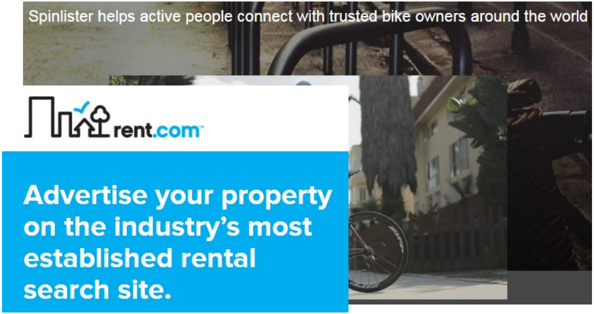

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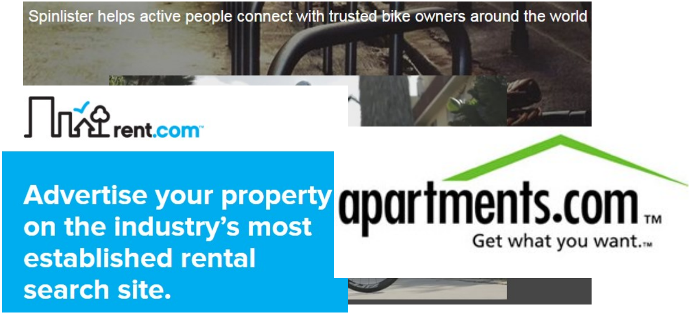

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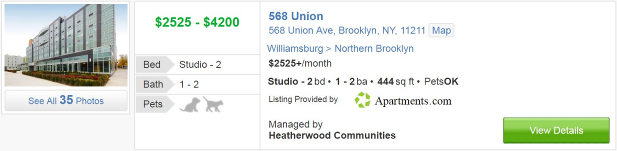

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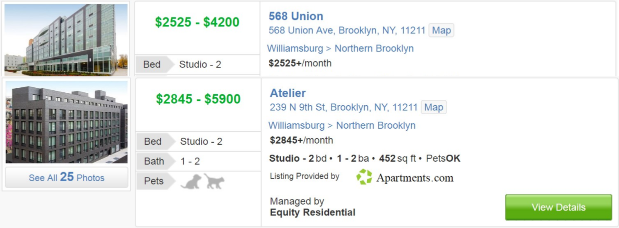

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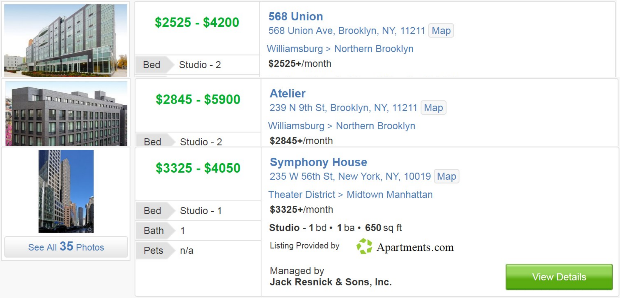

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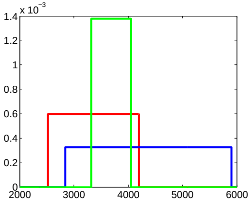

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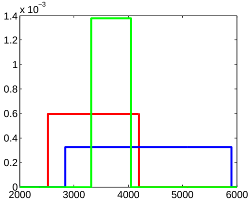


- With uncertain scores, a top-k query can only return an uncertain result.

# Related Work

- Soliman, Hyas and Ben-David [Soliman and Ilyas, 2009] study top-$k$ queries over objects with uncertain scores given as probability distributions.

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- In this paper, we consider **probabilistic top-$k$ queries** under the top-k semantics as in [Soliman and Ilyas, 2009].

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- (Objective:) **Probabilistic top-$k$ sequence**: the $\pi^{(k)}$ that maximizes $Pr(\pi^{(k)})$.

# Solutions

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- In this paper, we explore the variants of Markov chain Monte Carlo algorithms.

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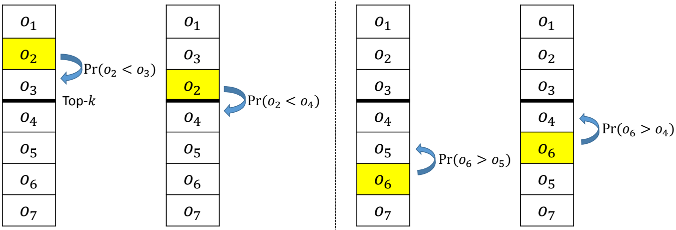

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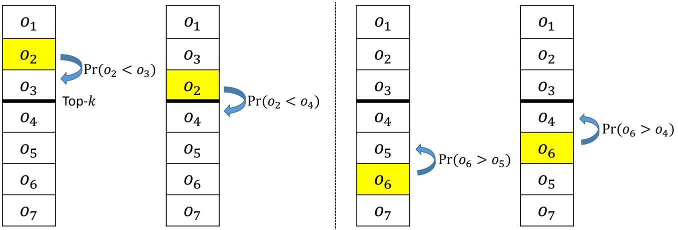


  - Acceptance Probability: $\alpha = \min(\frac{Pr(\pi_{t+1}^{(k)}) \cdot Pr(\pi_t | \pi_{t+1})}{Pr(\pi_t^{(k)}) \cdot Pr(\pi_{t+1} | \pi_t)}, 1)$

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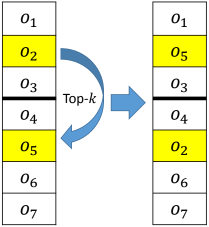

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    SwapEXP:

    $\alpha = \min\left(\frac{\widehat{Pr}(\pi_{t+1}^{(k)})}{\widehat{Pr}(\pi_t^{(k)})} = \exp(\beta(Pr(\pi_{t+1}^{(k)}) - Pr(\pi_t^{(k)}))), 1\right)$

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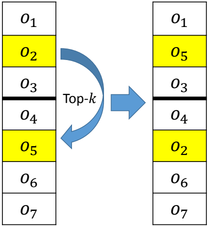

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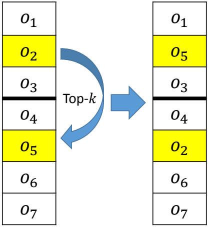


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- SwapEXP is more likely to reject the "worse" candidate state.

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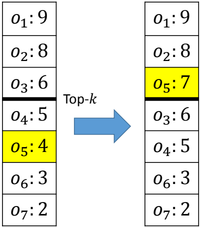

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`ReSampleEXP`:

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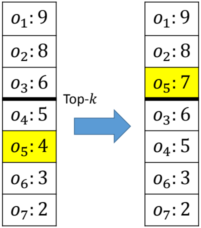

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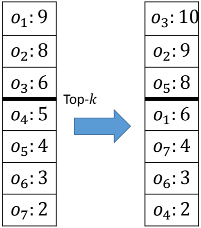

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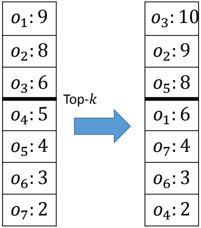

# Performance Evaluation

- Datasets: synthetic datasets

Table: Distributions

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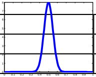
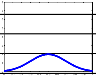
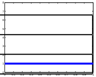
| | |  | |  | |  |
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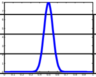
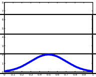
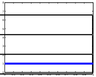
| median score | $G(0.5, 0.05)$ |  | $G(0.5, 0.2)$ |  | $U[0, 1]$ |  |
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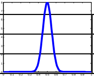
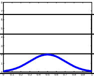
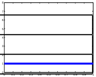
| median score | $G(0.5, 0.05)$ |  | $G(0.5, 0.2)$ |  | $U[0, 1]$ |  |
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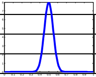
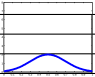
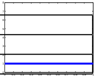
| | Setting 1 |  | Setting 2 |  | Setting 3 |  |
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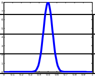
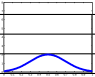
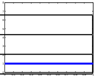
| median score | $G(0.5, 0.05)$ |  | $G(0.5, 0.2)$ |  | $U[0, 1]$ |  |
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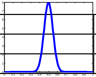
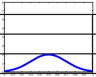
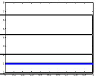
| | Setting 1 |  | Setting 2 |  | Setting 3 |  |
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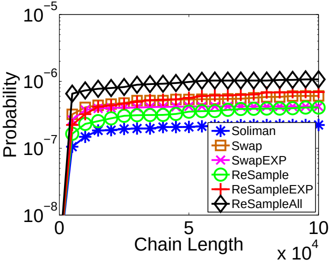
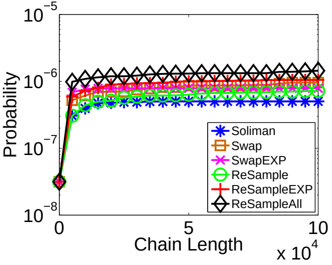
  - Probability of the Probabilistic top-$k$ sequence (higher $\rightarrow$ better)
  - Convergence of the Markov chains (Gelman-Rubin Convergence Diagnostic)
  - Efficiency (Complexity and runtime)

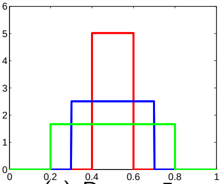
# Effectiveness of Six Algorithms (Probability)



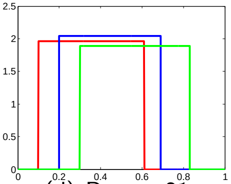

(a) Dataset5




(b) Dataset21

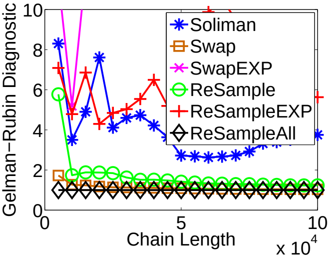


(c) Dataset5

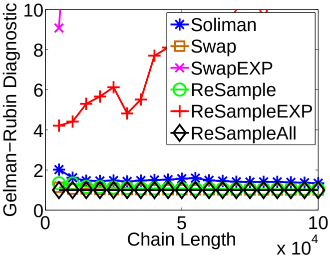


(d) Dataset21

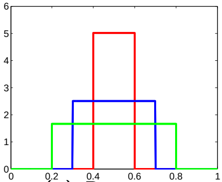
# Convergence of the Markov Chains



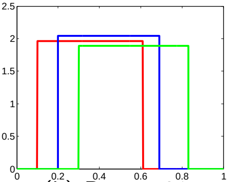

(e) Dataset5




(f) Dataset21




(g) Dataset5




(h) Dataset21

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# Efficiency

Table: Worst Case Time Complexity of Generating Next State

| | Soliman | Swap(EXP) | ReSample(EXP) | ReSampleAll |
|---|---|---|---|---|
| Time Complexity | $O(nk)$ | $O(1)$ | $O(n)$ | $O(nlogk)$ |

Table: Runtime Per Step of the Algorithms (seconds)

| | Soliman | Swap | SwapEXP | ReSample | ReSampleEXP | ReSampleAll |
|---|---|---|---|---|---|---|
| Runtime Per Step | 0.0058 | 1.9128 | 0.1163 | 0.0523 | 0.0071 | 0.9056 |

# Conclusion

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- We explore the design space for Metropolis-Hastings Markov chain Monte Carlo algorithms.
- We verify through extensive experiments that the proposed algorithms are more effective than the state of the art approach.
- `ReSampleAll` is the best, since it samples directly from the target distribution instead of depending on "local" information.

Thank you! Questions?

Top-k Queries

Uncertain Scores

MCMC

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# References I

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Ranking with uncertain scores.
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Soliman, M. A., Ilyas, I. F., and Ben-David, S. (2010).
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